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A Short Term Model of a Small Open Economy

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I. Introduction

A popular model of a small open economy is the one derived from the monetary approach to the balance of payments. As recognized by Johnson (1977) and lately by Frenkel et.al. (1980) to use the monetary approach to draw conclusions in situations in which the other variables that enter in the money market equilibrium equation (i.e. income, interest rates and prices) are changing, could result in wrong inferences. What is needed in those cases is a model in which changes in income, interest rate and prices are also accounted for.

The model that we will present below is an attempt to describe the short run equilibrium of a small open economy. In the present version it is an aggregate (one sector) model with one output and two assets (money and physical capital). The mo-

* I am greatful to Edgardo Barandiarán, Juan Eduardo Coeymans, Jorge Desormeaux and Andrés Solimano for comments on an earlier draft of this paper.

del is a mix of a monetary and a keynesian model. As a monetary model, it includes the important link between balance of payments and the money supply. It is a keynesian one, through the specification of the aggregate demand function and the determination of output by means of a flow equilibrium relationship in the market for output. It also includes a portfolio model for the determination of the equilibrium in asset markets. Elements of this model can be found in Turnovsky (1977),Dornbusch (1980) and Kingston and Turnovsky (1978). We also compare this model with one suggested recently by Frenkel et.al (1980) as a reconciliation of the keynesian and monetary model of the balance of payments. The model is only short run and therefore the important question of how the stock variables change is not dealt with here.

2. The Model

(1) $y = a(y, r - \pi^e) + g + nx(y, \frac{P^*e}{P}, y^*)$: Flow equilibrium in the output market (IS)

(2) $P = P(y)$: Aggregate Supply of output

(3) $qK^D = K^D(r, y, W)$: Domestic Demand for physical capital.

(4) $qK^F = K^F(r, r^*, \hat{e}^*, \rho, y^*, W^*)$: Foreign Demand for physical capital.

(5) $qK = qK^D + qK^F + qK^G$: Equilibrium condition in market for Physical capital.

(6) $q(r - \Pi^e) = C$: Rate of return on physical capital.

(7) $W = qK^D + \frac{M}{P}$: Wealth Constraint.

(8) $M = D + R$: Money Supply.

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(9) $\frac{M}{P} = L(r, y, W)$: Demand for Money.

Where:

y = real income.

a = real expenditure function.

r = nominal interest rate.

Π^e = Expected rate of inflation.

g = real government expenditures.

nx = net exports (real).

P^* = Foreign Prices.

P = Price of Domestic Output.

e = exchange rate.

M = Stock of Nominal Money.

D = Stock of nominal domestic credit.

R = Stock of International Reserves in domestic currency.

$\hat{e}^{+}$ = Expected rate of currency devaluation.

ρ = Country risk.

K = Capital stock.

q = Ratio of the market value to the reproduction cost of a unit of physical capital (Tobin's q).

C = Marginal efficiency of capital.

K^D = Physical capital owned by domestic residents.

K^F = Physical capital owned by foreign residents.

K^G = Physical capital owned by the government .

r^* = Foreign interest rate.

y^* = Foreign real income.

W = Domestic wealth.

W^* = Foreign Wealth.

Equation (1) is the IS for an open economy[1]. In this

[1] The model presented above can also be made compatible with a model in which we have two goods: tradables and nontradables. Tradables are separated into importables and exportables with the international terms oftrade between these two type of tradables fixed. Domestic consumption of the exportable is negligible and domestic production of the importable negligible. With that disaggregation we determine P from the model as stated. Using the value of P and a price aggregator function we solve for the price of non-tradables. The price of non-tradables is the adjusting endogenous price of the model.

equation[2] $\frac{\partial a}{\partial y} > 0$, $\frac{\partial a}{\partial (r-\pi^e)} < 0$, $\frac{\partial nx}{\partial y} < 0$, $\frac{\partial nx}{\partial e} > 0$, $\frac{\partial nx}{\partial P} < 0$.

Equation (2) is a short run aggregate supply equation. It can be derived from equilibrium in the labor market for a given money wage (Modigliani (1963)); equilibrium in the labor market with different price expectations for demanders and suppliers of labor (Branson (1978)); from staggered wage contracts (Taylor (1980)) or from price surprises for given expectations (Mc Callum (1980)). In this equation $\frac{\partial P}{\partial y} > 0$.

Equation (3) is the domestic demand for physical capital. We have assumed that no interest is paid on holding money. Thus, r is the difference between the real return on capital stock $r-\pi^e$ minus the real return on money $-\pi^e$ (Tobin (1969), Sargent (1979)).

Equation (4) is the foreign demand for physical capital.

Equation (5) is the equilibrium condition in the market for physical capital.

Equation (6) is the equation for the rate of return on physical capital. In this equation we have assumed that capital

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2 The last two inequalities follow from assuming that the Marshall-Lerner conditions are satisfied.

has an infinite life and a constant marginal efficiency.(Tobin (1969)).

Equation (7) is the domestic wealth constraint.

Equation (8) is the money supply equation, assuming that only the central bank issues money.

Equation (9) is the demand for money function. (Goldfeldt (1973), Laidler (1980), Corbo (1981 a) and (1981 b)). If equations (3) and (7)are satisfied then by Walras law equation (9) is also satisfied. Thus, we will not use equation (9) when we solve the model in the next section.

We will treat this model as a system of 8 equations in 8 endogenous variables: y, r, P, M, K^D, K^F, q and R. Our exogenous variables are g, K^G, W, K, Π^e, P^*, e, D, r^*, y^* and W^*.

3. The Solution of the Model

We will assume that a solution to this model exists[3] and we will solve the model reducing it first by substitution.

We start by substituting (3), (4) and (6) in (5) obtaining the following locus in r and y for which the market for physical capital is in equilibrium.

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[3] A sufficient condition for a unique local solution to exist is that the Jacobian of the system has to be non-zero. A sufficient condition for a unique global solution can be derived in terms of properties of the Jacobian matrix. For our models, which are an extension of the IS-LM framework global uniqueness is guaranteed.

$$(10) \quad \frac{C}{r-\pi^e}(K - K^G) = K^D(r, y, W) + K^F(r, r^*, \hat{e}^+, \rho, y^*, W^*) + K^G$$

Differentiating this equation totally with respect to r and y we obtain:

$$-\frac{C\ K}{(r-\pi^e)^2}\,dr = \frac{\partial K^D}{\partial r}\,dr + \frac{\partial K^D}{\partial y}\,dy + \frac{\partial K^F}{\partial r}\,dr$$

$$\frac{dr}{dy} = \frac{-\dfrac{\partial K^D}{\partial y}}{\dfrac{\partial K^D}{\partial r} + \dfrac{\partial K^F}{\partial r} + \dfrac{C(K - K^G)}{(r-\pi^e)^2}}$$

Thus, to obtain the sign of $\frac{dr}{dy}$ we need to know the signs of $\frac{\partial K^D}{\partial y}$, $\frac{\partial K^D}{\partial r}$ and $\frac{\partial K^F}{\partial r}$. We assume $\frac{\partial K^F}{\partial r} > 0$. For the signs of the other partial derivatives we have to use the private sector wealth constraint.

The private sector wealth constraint is given by

$$W = L(r, y, W) + K^D(r, y, W)$$

Differentiating this equation totally we obtain:

$$dW = \left(\frac{\partial L}{\partial r} + \frac{\partial K^D}{\partial r}\right) dr + \left(\frac{\partial L}{\partial y} + \frac{\partial K^D}{\partial y}\right) dy + \left(\frac{\partial L}{\partial W} + \frac{\partial K^D}{\partial W}\right) dW$$

This equality holds for all values of r, y and W if and only if

$$\frac{\partial L}{\partial r} + \frac{\partial K^D}{\partial r} = 0$$

$$\frac{\partial L}{\partial y} + \frac{\partial K^D}{\partial y} = 0$$

$$\frac{\partial L}{\partial W} + \frac{\partial K^D}{\partial W} = 1$$

We assume $\frac{\partial L}{\partial r} < 0$, $\frac{\partial L}{\partial y} > 0$, $1 > \frac{\partial L}{\partial W} > 0$ and therefore we need $\frac{\partial K^D}{\partial r} > 0$, $\frac{\partial K^D}{\partial y} < 0$ and $0 < \frac{\partial K^D}{\partial W} < 1$ to satisfy the above restrictions.

Replacing these signs in the equation for $\frac{dr}{dy}$ we find $\frac{dr}{dy} > 0$. Equation (10) is represented by the line KK (equilibrium in the market for physical capital) in the diagram below. If we have perfect capital mobility $\frac{\partial K^F}{\partial r} = \infty$ and thus the KK locus is horizontal. The KK locus is also horizontal if $\frac{\partial K^D}{\partial r} = \infty$, that is if money and physical capital are perfect substitutes.

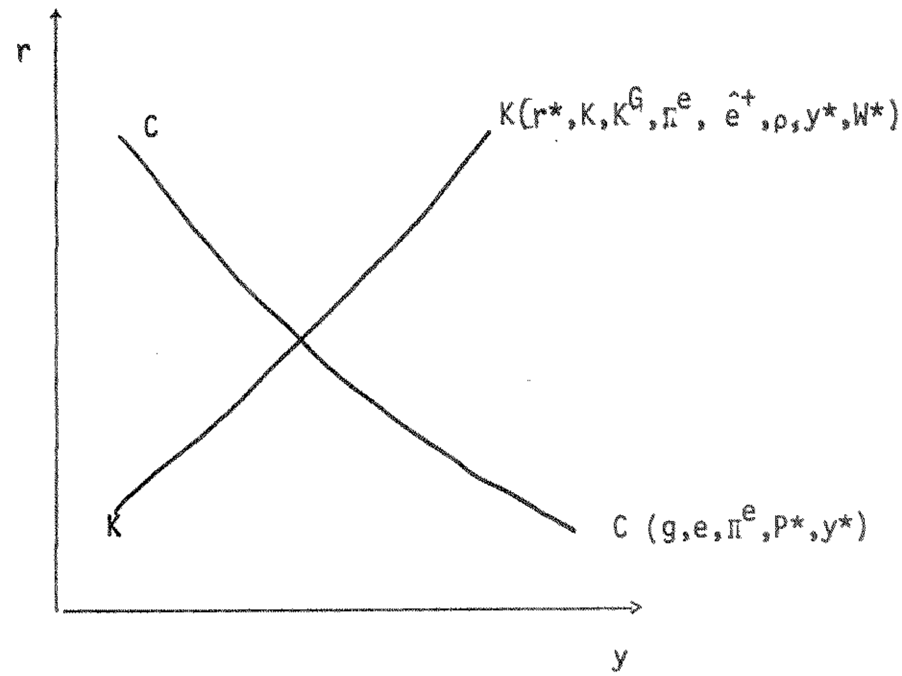

Figure 1

From equation (2) we have $P=P(y)$, we replace P in equation (1) and we obtain a locus between r and y for which the commodity market is in equilibrium. We call this locus CC

$$(11)\quad CC(r,\ y;\ g,\ e,\ \Pi^e,\ P^*,\ y^*) = 0$$

The slope of this locus is negative[4]. For a keynesian model with an infinite elastic supply ($\frac{\partial P}{\partial y} = 0$) the CC functions is flatter but also with a negative slope. In contrast, if the supply function is completly inelastic, then the CC function is vertical.

Finally for the values of y and r that clear the commodity and capital markets, and the value of qK^D from (3) we solve (7) for $\frac{M}{P}$. With this value of $\frac{M}{P}$ the value of P from equation (2) and given D we solve equation (8) for the stock of international reserves in domestic currency (R).

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[4] From (1) we obtain

$$dy = \frac{\partial a}{\partial y} + \frac{\partial a}{\partial (r-\Pi^e)} dr + \frac{\partial nx}{\partial y} dy + \frac{\partial nx}{\partial \frac{P^*e}{P}} \frac{\partial \frac{P^*e}{P}}{\partial P} \frac{\partial P}{\partial y} dy$$

$$\left[1 - \frac{\partial a}{\partial y} - \frac{\partial nx}{\partial y} - \frac{\partial nx}{\partial \frac{P^*e}{P}} \frac{\partial \frac{P^*e}{P}}{\partial P} \frac{\partial P}{\partial y}\right] dy = \frac{\partial a}{\partial (r-\Pi^e)} dr$$

But

$$1 - \frac{\partial a}{\partial y} - \frac{\partial nx}{\partial y} > 0 \text{ and } \frac{\partial nx}{\partial \frac{P^*e}{P}} > 0, \frac{\partial \frac{P^*e}{P}}{\partial P} < 0 \quad \text{and}$$

$\frac{\partial P}{\partial y} > 0$ (from supply function)

Thus, the term in square bracket is positive.

4. Comparative Statics

Now we will use the model of the previous section to make some exercises in comparative statics. We will study the effect of an increase in the foreign rate of interest (r^*), increase in the exchange rate (e) and decrease in world income (y^*).

Before proceeding to do these exercises we want to say a couple of words about dynamics. We assume that $\dot{y}=\theta_1 EDY$ and $\dot{r} = \theta_2 ESK$, where a dot over a variable indicates time derivatives, EDY and ESK are the excess demand for output and excess supply of capital respectively. If θ_1 and θ_2 are positive parameters, then the system is stable. The arrows in Figure 2 indicate the direction of the changes in r and y. In deriving this dynamic adjustment, we observe that to the right of the locus KK we have excess supply of capital (remember that $\frac{\partial K^D}{\partial y} < 0$), and therefore r has to increase to clear the capital market. To the left of KK we have excess demand for capital, and by the same argument r has to decrease to clear the capital market. To the right of the locus CC we have excess supply of output, and thus y has to decrease to clear the commodity market. Also, to the left of the locus CC we have excess demand for output and thus y has to increase.

Now we will perform some excercises in comparative statics.

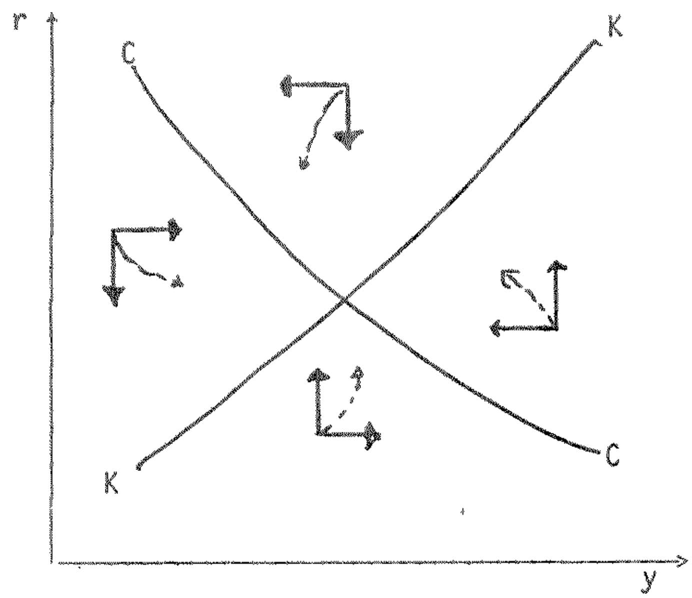


Figure 2: Dynamic Adjustment

a) Increase in the foreign rate of interest.

In this case, at the same domestic rate of interest (r) we will have a lower quantity demanded of physical capital by foreigners. The KK curve shifts to the left,and we will have a new equilibrium with a higher interest rate and a lower level of income.

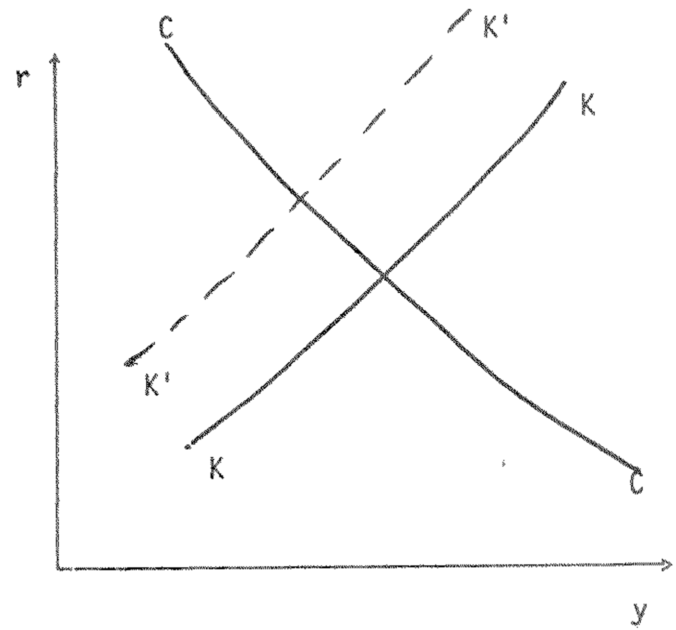


Figure 3: Increase in r*

What happens with the balance of payments?

The lower income and higher interest rate imply from equation (3) a higher qK^D. The higher qK^D and lower price level (from equation 2) imply from equation (7) a lower stock of nominal money. Using equation (8), for a given D we need a lower level of reserves to equilibrate the money market.

We get the same qualitative result if the shift is due to an increase in $\hat{e}^+$ or in ρ.

b) Increase in the Exchange rate.

If there is an increase in the exchange rate (with $\hat{e}^{+}$ and the other exogenous variables of the model constant) then the CC function shifts to the right we will have a new equilibrium with higher y and higher r in Figure 4.

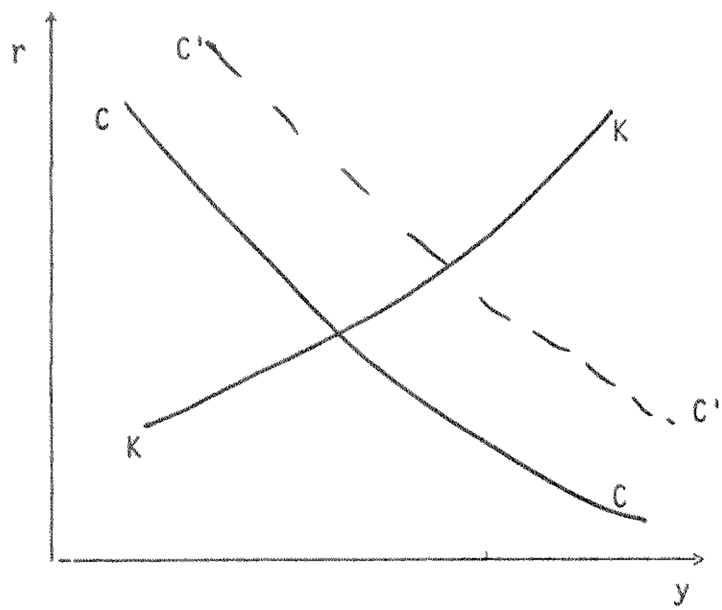


Figure 4: Increase in e

c) Decrease in World Income.

For a decrease in world income the CC function and the K K function shift to the left. We will have a new equilibrium with a lower income,but the new interest rate can be higher or lower depending on the relative shifts of both functions.

5. Some Extensions of the Basic Model

In this section we will extend the model of section 2 introducing the government budget constraint. In this model we will analyse the effects of alternative fiscal and monetary policies.

Now in the aggregate demand function we have to include taxes to obtain disposable income. We also need to include a relation between ΔD, ΔK^G and the government budget.

Specifically, we add to our model the following equations.

(12) $g + q \cdot \Delta K^G = t + rqK^G + \frac{\Delta D}{P}$

(13) $t = \tau_0 + \tau_1 \cdot y$

Where the new symbols introduced are:

t = real taxes.

τ = marginal tax rate

Introducing (13) in (1) we rewrite the aggregate demand function as:

(1)' $y = a(y, r-\Pi^e, \tau_0, \tau_1) + g + nx(y, \frac{eP^*}{P}, y^*)$

We work now with the system of equations (1)' and (2) to (8) and we use equation (12) to analyze how the budget is financed.

a) Increase in domestic credit.

Given the government budget constraint to analyze the effect of an increase in domestic credit,we need to analyze which of the other terms in equation (12) changes. We will analyze an increase in domestic credit utilized to buy physical capital(pure monetary policy). Thus, $\Delta D = qP\Delta K^G$. In equation (1)' nothing changes with the increase in domestic credit. Thus, the CC function does not change. In the market for physical capital there is an increase in the quantity demanded and thus the KK function shifts to the right.

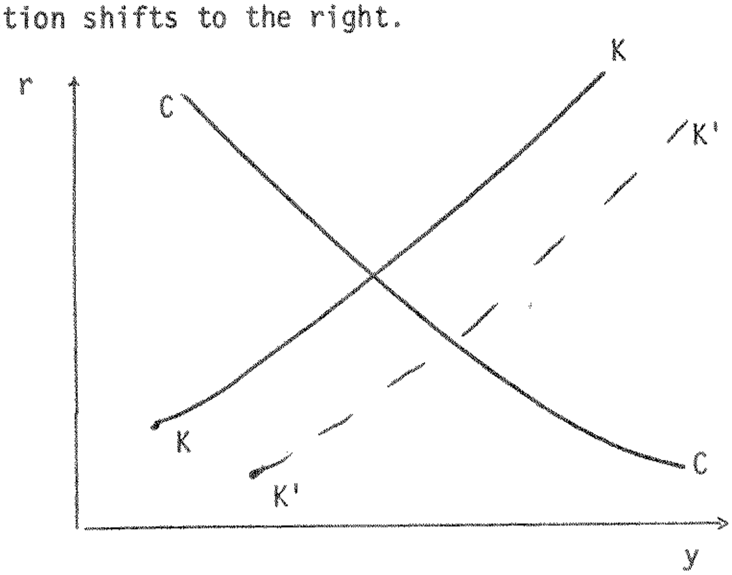


Figure 5: Increase in D

In the new equilibrium we have a lower r and a higher y. Then by equation (3) we will also have a lower qK^D. From equation (7) we will have in the new equilibrium a higher $\frac{M}{P}$. From equation (2) we will have a higher P. Thus, M in the new equilibrium will be higher. In the second period further changes in ΔD and ΔK^G will take place to satisfy (12).

b) Decrease in government taxes.

We will assume that the government sells part of his stock of physical capital to finance a tax reduction (like reducing social security taxes).

For simplicity we will assume in equation (1)' that the change take place as a reduction in τ_0 with $\Delta\tau_0 = \Delta qK^G$. In equation (1)' the only change is an increase in absorption (we assume $\frac{\partial a}{\partial \tau_0} < 0$). Thus, the CC function shifts to the right. The reduction in government holding of physical capital shifts the KK function toward the left.

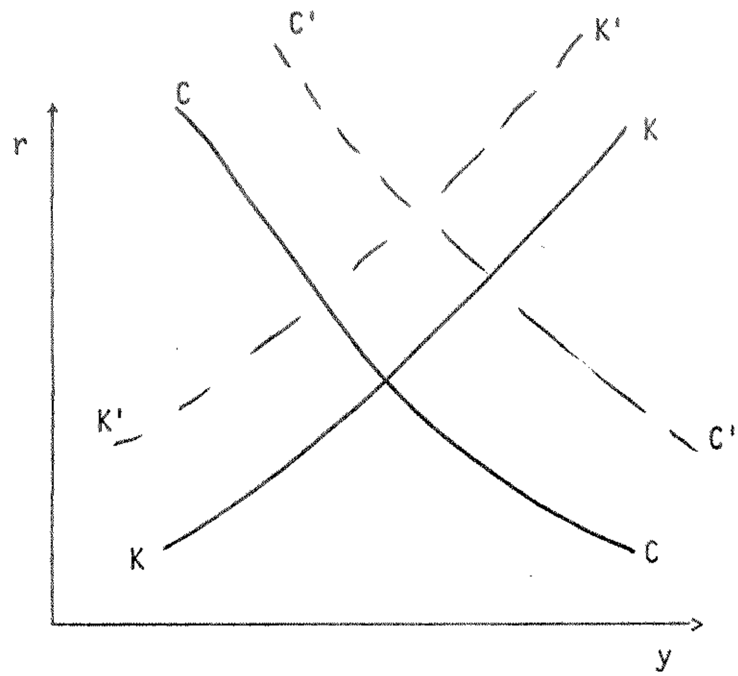


Figura 6: Decrease in Taxes Financed Selling Physical Capital.

In the new equilibrium we will have a higher interest rate,but the direction of the change in income can not be obtained from geometry alone. In the second period, further changes will take place in K^G and/or D to satisfy (12).

5. <u>An Alternative Formulation of Capital Flows</u>.

Instead of using the modern portfolio model for stock of foreign capital,we can go back to the Mundell model in which capital flows are made a function of interest rate and income. In this model we have full compatibility between the mone tary approach to the Balance of Payments and the Keynesian Balance of Payments. This model can be found in Turnovsky (1977) and in Frenkel et.al. (1980).

The new model is given by the following equations:

(1) $y = a\,(y, r - \Pi^e) + g + nx(y, \frac{P^*e}{P}, y^*)$

(2) $P = P(y)$

(8) $M = D + R$

(9) $\frac{M}{P} = L(r, y, W)$

(14) $R = P \cdot nx(y, \frac{P^*e}{P}, y^*) + F(y, r, r^*, \rho)$ + R-1

Where $F(y, r, r^*, \rho)$ is the net capital inflow.[5] Equation (14) is a keynesian balance of payments equation.

The system of equation (1), (2), (8), (9) and (14) is a system of 5 equation in five unknowns y, r, P, M and R.

Now the system is not separable, thus, we need to solve the three markets simultaneously.

We can solve the model again by substitution. We substitute (2) in (1) obtaining a locus on y and r. We can write this locus as

[5] Here capital flows are a function of the levels of y, r and r*. In the model of the previous sections capital flows were a function of the change in the levels of the above variables. We assume that $\frac{\partial F}{\partial y} > 0$ and $\frac{\partial nx}{y} + \frac{\partial F}{\partial y} < 0$.

(15) $r = r\,(y;\ g,\ \Pi^{e},\ P^{*}\ ,\ e),\ \ \frac{\partial r}{\partial y} < 0$

By substituting into equation (9) the variables P and r from equation (2) and (15), we obtain:

$$M = P(y)\ L(r\ (y;\ g,\ \Pi^{e},\ P^{*},\ e), y,\ W)$$

substituting this expression in (8) we obtain the money market equilibrium condition,which form the core of the monetary approach to the balance of payments.

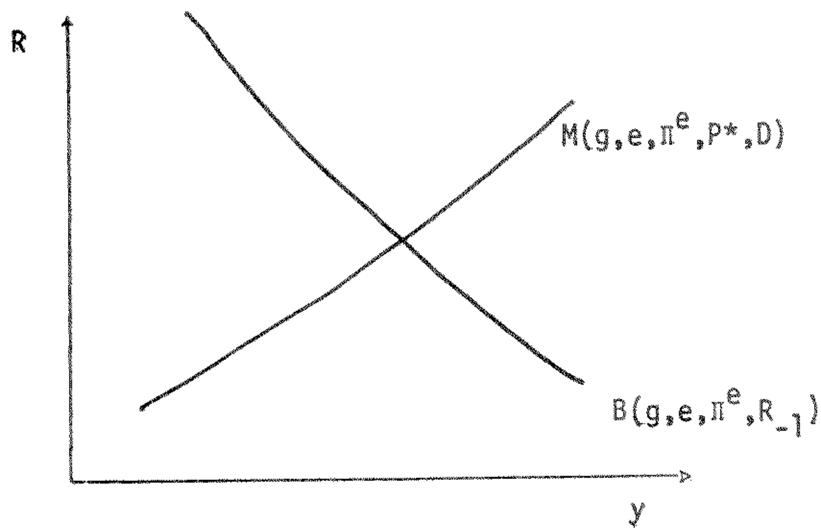
(16) $R = \ P(y)\ L(r(y;\ g,\ \Pi^{e},\ P^{*},\ e)\ y,\ W)-D$

Now we go back to the keynesian balance of payments equation. We replace equations (2) and (15) in equation (14) obtaining:

(17) $R = P(y)\ nx(y,\ \frac{P^{*}e}{P}, y^{*}) + F(r(y;\ g, \Pi^{e},\ P^{*},\ e),\ y,\ r^{*},\ \rho)$

$+ R_{-1}$

Equations (16) and (17) form a system of 2 equations in 2 unknowns R and y. These two equations are represented in the diagram below.



In this diagram a decrease in capital inflows for a given R and y shifts the B curve to the left obtaining a new equilibrium with lower R and y. Replacing in (15) we obtain also a higher r.

The study of the comparative static of this model can be found in Turnovsky (1977, chapter 9) and in Frenkel et. al. (1980). The study of the stability of this model can be found in Turnovsky (1977, Ch.9).

Final Comments

The model developed here pretends only to iluminate macro general equilibrium aspects, a richer analysis of the real structure of an open economy can be obtained from a two sector model. For this purpose one can use the standard Swan - Salter-Corden - Dornbusch Australian Model (Dornbusch (1980, Ch.6), Salter (1959)).

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