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Neuronal Utility and Choice by an Organism

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Neuronal utility and choice by an organism[1]

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Abstract

This paper presents a theory of choice that yields a set of new predictions about behavior when compared to the expected utility model, encompassing the existence of automatic responses and attentional biases. A formula comparable to the one of expected utility is identified. Under certain restrictive conditions identified here, the aggregate behavior of this brain model is equivalent to the one proposed by the expected utility model.

This theory of choice is based on the concept of a race between different processes that occur in parallel inside the brain of an organism, run by different groups of neurons. Action is defined by the outcome of this competition, which is won by the process that is completed first. The time it takes for one process to finish the race is a function of the desirability it evokes, but additionally depends on the length of the neuron chain and how the sensory system reacts to information.

JEL codes: D87, D81

Keywords: Expected Utility, Neuroeconomics, Decision-making, Economics of neural activity, Cognitive Psychology.

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1. Introduction

In classic expected utility models, decisions are made on the basis of preferences, expectations, and objective restrictions. Preferences are the core of these models: they are unique (which assures coherence), fixed at the moment of choice (but may change over time), and are independent of both restrictions and the form in which information is obtained. Therefore, the agent´s method for decoding information is irrelevant.

Nonetheless, a growing consensus in neuroscience suggests that the brain makes decisions by following a cycle that starts with the construction of a representation of the problem and the potential actions that could be followed. Later, the courses of actions under consideration are evaluated and one of these is selected. Finally, the outcome observed after the implementation of the selected action is used to improve the quality of future decisions (see Rangel et al, 2008) for a review). Moreover, evidence shows that information processing takes time and that different brain areas receive and send information at different moments and with varying intensity. This paper takes this cycle as given.

We study the implications of information decoding on the decision-making process. The paper starts by modeling brain cells and how they share information. Then, we construct a decision-making cycle according to the evidence found by neuroscientists. Next, we derive choice in terms of a competition between brain processes. Finally, by simplifying a few assumptions, we reach a formula that describes decision-making, the equivalent of a utility model.

The main result is that choice can be modeled without assumptions such as the maximization principle and the standard preference axioms, which presume a single decision-making entity. Instead, choice emerges as the result of the brain´s evaluation process, whereby chains of cells exchange and process information until a decision is reached. Moreover, we show that the combined behavior of brain cells can be presented in a model that is similar to but more flexible than expected utility models.

In this model, it is useful to imagine choice as a competition between brain processes whereby several possible actions are evaluated in parallel by different groups of neurons. That is, the action that completes its process first is executed. The overall processing time of this action is composed of the time needed to characterize the current situation and propose this action, and the desirability this action evokes. This desirability has a weight, which might be given by experience or education. If one action is considered to be more desirable than another by a group of brain cells, it is processed more rapidly and thus has a better chance of being executed. This conceptualization allows modelling the following two possibilities:

a) If a brain structure uses shortcuts in some situations, the actions evaluated by that structure will have higher chances of execution.
b) If the attention of some group of brain cells is drawn to some attributes instead of others (because the perception system focuses on differences instead of levels, because the brain is hardwired to look for dangerous situations, or any other reason) decision-making will be altered.

In other words, desirability or reward is not the only source of competitiveness between possible actions. Context also influences whether one process or another's action signal finishes first and is executed. Therefore, some decision-maker's actions will be inconsistent with the standard preference axioms if context changes the group of neurons that control actions.

Unlike other models that depict conflicts of choice inside the brain, our model resolves this struggle through a new variable: the time it takes to process information for each of these preferences (held by different groups of neurons). This means that when the activity or speed of one brain process increases above a certain threshold, it will make the decision even if it is not as fine-tuned to the environment as other processes. In this setting, errors of judgment could be defined as the posterior belief by a brain that the criteria she used was incorrect. That is, after executing an action, the same person (other groups of neurons in her brain) could evaluate her decision-making process as biased, myopic, or even unreasonable, because at the time of decision-making a different group of neurons won the competition to decide which action to take. It is perhaps important to mention that decisions may also be evaluated ex –post as underperforming if the time needed to make that decision was too long.

The race between processes needs to be regulated in the long run. If one process executes an action rapidly, but the consequences of those actions affect the organism's survival, its prevalence should be reduced. Nevertheless, an optimally attuned brain will not necessarily be composed of a unique decision-making process. Preserving both fast and slow processes may be consistent with an evolutionary perspective of decision-making because the organism's survival rate may be linked to its reaction speed and ability to anticipate dangerous situations. Since brains need time to process information, and dangerous situations become increasingly harder to anticipate in a competitive environment, an organism could benefit by preserving several brain processes. Nevertheless, if groups of processes coexist they could be pondered –perhaps through experience or education- to be more effective. The model in this paper simplifies this issue by giving each process an exogenous weight, possibly determined by experience and education. That feedback loop, however, is not modeled here.

Even though this model assumes that cells act independently, we find that under certain restrictive circumstances, our model of decision-making reduces to a model in which a single agent optimizes according to a coherent preference ordering. In other words, some micro processes result in the appearance of macro maximization. This is not uncommon in science. One analogy is provided by an economy where each agent and firm chooses independently their consumption and production respectively. If all markets are perfectly competitive, the economy allocates resources overall as if a maximizing agent were present, namely a benevolent planner, as suggested by Adam Smith's intuition of a "benevolent hand", and as proven rigorously by Arrow and Debreu (1954). Another analogy comes from the Lagrangian formulation of classical mechanics, which models the behavior of conservative mechanical systems as if total energy were being minimized by somebody. A further analogy is provided by evolutionary theories, where some selection processes yield organisms that are almost optimally adapted to their environment, as if someone had "designed" them.

The next section presents the simplified mathematical formulation of our model, describes intuitively two situations in which expected utility comes up short as an explanation for behavior, and presents an application to finance. Section 3 reviews the literature on the behavior of brain cells and neurons that is pertinent to the mechanics of choice in terms of processes, and the macro view of decision-making, covering both theory and experiments. Section 4 presents the model in two main steps: first, a general model of a brain cell is presented, and then the model is specialized into three different types of cells, to which a general class of interconnection is added, in order to mimic the three stages of the decision-making cycle, in line with the biological evidence. Afterwards, it is shown that the previous cycle can be interpreted as the competition between processes to execute their actions. Later, a macro decision-making model is presented and obtained and a set of conditions under which the aggregate behavior of the model is equivalent to the one predicted by expected utility theory is found. The mathematics are developed in Appendices. Section 5 summarizes and concludes.

2. The model, two insights and one example

Later on, section 4 models how information sharing at the cellular or neural level can contribute to a decision between two or more alternatives. By following this chain of processes, that section finds an equation describing decision-making by the organism. This section presents a simplified version of that result, with the aim of building intuition and presenting simple applications.

In this simplified setting, the action a^* executed by the organism is the one that takes the least time $t^{p,a}$ to process among several processes p in competition:

$$a^* = arg\min_{(p,a)} \quad t^{p,a} = \frac{N^p}{s_p} + \frac{[1/V_p]}{m_p \cdot \left(\sum_c \pi_p(a,c) \cdot u_p(a,c)\right)} \qquad \text{(i)}$$

$$where \quad V_p \equiv \sum_k w_{pk} \cdot v_k^0$$

where c is the posible consequence of executing action a, $\pi_p(a,c)$ is the equivalent of the subjective probability that links action to consequence in process p and $u_p(a,c)$ is the feeling of pleasure felt by those brain cells that participate in process p when valuing consequence c. The number of processes that compete is given by P $(p = 1,2, \dots P)$.

In each process p, N^p is the number of brain cells needed to decode and evaluate information (a measure of the simplicity of the process, which may run from reflexes to slow maturation), s_p is the average speed of processing (a measure of the complexity level of the actions being evaluated), V_p is the weighted average of the intensity of the stimuli provided by the sensory organs (which are denoted v_k^0 and are weighted by the w_{pk}), and m_p reflects the overall weight that process p has earned in this organism's decision making (perhaps through experience and education).

As can be observed, the equivalent to expected utility used by each process is $\sum_c \pi_p(a,c) \cdot u_p(a,c)$. One novelty is that other factors, denoted by N^p, s_p, V_p and m_p, are shown to influence decision making as well. The formula indicates how to combine these to represent decision-making by the organism.

Figure 1 shows how this choice model can be interpreted in terms of a race. Time is in the horizontal axis. The time needed to reach the ceiling (the finishing line) is determined by the date at which a process starts running (the size of t_{0p}) and by the overall speed of the process (the slope), which is $V_p \cdot m_p \cdot \left(\sum_c \pi_p(a,c) \cdot u_p(a,c)\right)$. As (i) shows, one process that starts with a lead time disadvantage will be completed before the other only if its speed is able to overcome the lead time disadvantage.

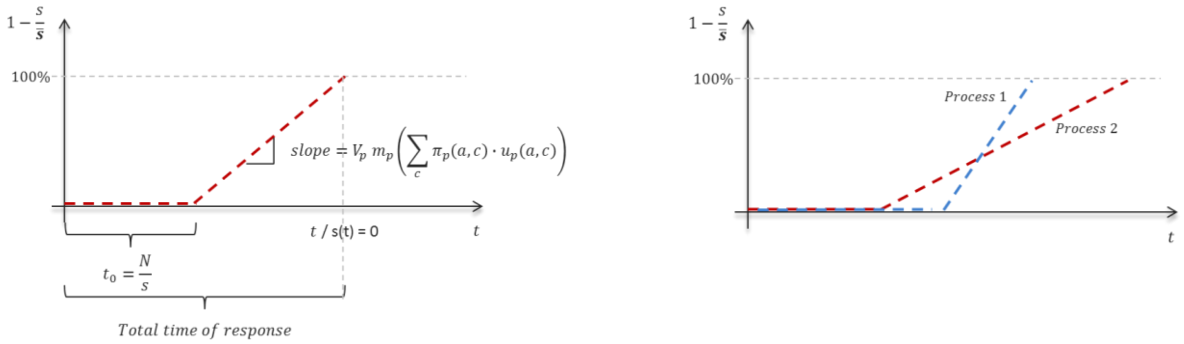

Figure 1: Graphic representation of decision-making as a race between two processes.

Note that the overall time needed by each process to reach the ceiling is not homothetic with respect to the overall speed.[4]

To simplify, this introductory section assumes that there are only two representative processes: a semi-automatic, hard-wired process E that uses heuristics to produce a fast response (such as the fight of flight response, which allow us to act quickly in difficult situations), and a rational process R that takes into account most possible future scenarios and weighs complex factors and possibilities more carefully, but is slower. The “E” in the semi-automatic process stands for strong emotions such as Anger and Fear. Of course, competitive processes in humans are likely to combine emotions and rationality.

In the two-process case, process E will dominate the organism’s action, when its processing time is shorter than for the rational process, i.e. if:

$$t^{E,a_E^*} \leq t^{R,a_R^*} \tag{ii}$$

Rewriting (ii) in terms of intensity V_p, using (i), the threshold that the emotional process must overcome in order to win the race is found to be the right hand side of (iii):

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[4] If both overall speeds in the right-hand side diagram are multiplied by a large enough number, making both upward-sloping lines almost perpendicular to the time axis, process 2 reaches the ceiling before process 1 because the difference in lead times dominates.

$$V_E \equiv \sum_k w_{Ek} \cdot v_k^0 \geq \frac{m_R}{m_E} \cdot \frac{\frac{U_{R,a_R^*}}{U_{E,a_E^*}}}{\frac{1}{V_R} + \left(\frac{N^R}{s_R} - \frac{N^E}{s_E}\right) \cdot m_R \cdot U_{R,a_R^*}} \qquad \text{(iii)}$$

where U_{p,a_p^*} with $p = E, R$ is a shorthand for the utility achieved when process p wins the race ($U_{p,a_p^*} \equiv \text{argmax}_a \sum_c \pi_p(a,c) \cdot u_p(a,c)$).

The threshold in the right-hand side of (iii) yields the following insights:

a) If the organism learns to trust less its emotional process (if m_R / m_E increases) then it will be harder for the emotional process to win the race;
b) If the organism's relative reward to acting rationally rises (if ($U_{R,a_R^*} \,/\, U_{E,a_E^*}$) rises), the threshold rises, so the emotional process and is less likely to win the race;
c) If the organism's attention is drawn towards the rational process and its intensity rises (V_R increases), then the threshold rises and the emotional process is less likely to win the race;
d) If the level of complexity increases, so the time difference $\frac{N^R}{s_R} - \frac{N^E}{s_E}$ needed to understand the situation rationally rather than emotionally, then the speed of decisions increase in importance, the threshold falls and the emotional process is more likely to win the race;
e) If both processes yield similar outcomes and danger is considered low (if $U_{R,a_R^*} \,/\, U_{E,a_E^*}$ remains constant and U_{R,a_R^*} rises), then the organism relaxes and the threshold falls, so the faster process (emotional) is more likely to win the race; and
f) If the organism's absolute trust in both process rise proportionally together (if m_R / m_E remains constant and m_R rises), then the organism relaxes and the threshold falls, so the faster process (emotional) is more likely to win the race.

In order to build intuition, this threshold is applied now to three examples and applications.

2.1. Irrational thoughts may hijack decision-making

Consider the Scorpion and the Frog fable:

> *A scorpion and a frog meet on the bank of a stream and the scorpion asks the frog to carry him across on its back. The frog asks, "How do I know you won't sting me?" The scorpion says, "Because if I do, I will die too."*
> *The frog is satisfied, and they set out, but in midstream, the scorpion stings the frog. The frog feels the onset of paralysis and starts to sink, knowing they both will drown, but has just enough time to gasp "Why?"*
> *Replies the scorpion: "It is my nature..."*
>
> *The Scorpion and the Frog, Aesop's fable*

When the scorpion is on the shore, it acknowledges that killing the frog while in the stream will lead to its own death. Nevertheless, when the scorpion is above the frog, an automatic process, hard-wired into its brain, hijacks decision-making and results in a sting that is fatal for both animals. The scorpion's rational process cannot prevent its nature from acting.

If reflexes use a shorter neuron chain to reach an action proposal than a rational process does, then reflexes have an edge over rational processes in certain situations. The parameter in the model that captures this example is the difference in the number of cells involved in each process of representation ($N^R \gg N^E$). As can be seen in the equation (iii), if the difference between these numbers of cells rises, the threshold needed for the hard-wired automatic process to control actions will decrease. If the difference is sufficiently large, a signal from the sensory system (such as detection of a sure opportunity to kill the frog) allows the hard-wired process to hijack decision-making.

2.2. An automatic sensory system changes how much attention an alternative gets

The model allows the sensory system to favor some alternatives with respect to others, influencing decision-making as in the following example:

> *Once when Jacob was cooking some stew, Esau came in from the open country, famished. He said to Jacob, "Quick, let me have some of that red stew! I'm famished!"*
> *Jacob replied, "First sell me your birthright."*
> *"Look, I am about to die," Esau said. "What good is the birthright to me?"*
> *But Jacob said, "Swear to me first." So he swore an oath to him, selling his birthright to Jacob.*
> *Then Jacob gave Esau some bread and some lentil stew. He ate and drank, and then got up and left.*
> *So Esau despised his birthright.*
>
> *Genesis 25, The Bible, New International Version*

When Esau is hungry and smells food, he cannot restrain himself from choosing food over his birthright. In the model, the smell of the stew (sensory inputs) interacts with context (Esau is hungry) to provide additional intensity and thus speed to one of the processes, which we may call impulsive and label E. In the absence of that context (no hunger), speed would not increase. In the model presented above, this can be captured by a higher value for w_{Ek}, the weight that react to outside-world information v_k^0, due to hunger. This raises the intensity V_E for process E and overcomes the threshold.

One interpretation is that the attention system, which receives inputs and automatically sends signals to the brain, may have evolved by adapting to a different environment than today's. This may explain why its signals may bias decision-making towards actions that are less desirable in the modern environment. Another interpretation is that decision making is affected by the amount of attention an alternative receives. Since the automatic sensory system ponders how much weight to assign to each sensory input, decision making may be consistently biased if this system is not adequately trained.

A testable prediction of this aspect of the model is that impulsive actions are taken faster than reflexive ones in a specific context (when hungry, for example), and not in others.

2.3. Where do "noise traders" come from?

In the last decades, several authors have identified and modeled financial markets where a group of investors buy or sell assets according to reasons different from "fundamental" data (see Barberis &

Thaler (2003) for a review). Individuals who belong to this group have been variously described as having poor timing, being partially informed, over or under react, or behaving randomly. They are called "noise traders". However, these actions should lead to losses (on average) or to the progressive loss of weight in market (be it because they own less capital or because they delegated their portfolio management). Nevertheless, several authors have shown that noise traders remain.

Now we use the simplified model of this section to suggest that many professional investors may, in some context, become noise traders. The model also identifies conditions that change the number of traders acting as noise traders, allowing predictions that may be tested in the future.

When the emotional process is active, trading is subject to emotions such as anger, fear, disgust and so on. On the contrary, when the rational process is dominant, trading can be assumed to take into account market information dispassionately. The variables that influence the threshold in equation (iii) apply here as well.

Now consider an asset market that is informationally fully efficient, in the sense that prices reflect all available information. In such an informationally efficient market, noise traders suffer no loss. In terms of the simple model here, this implies that the ratio of reward between applying a rational thought process to choose trades, and using a random device such as throwing a dart to choose trades and portfolios, (i.e. becoming a "random noise trader"), will be one ($U_{R,a_R^*} = U_{E,a_E^*}$). Since rational thought takes more time to understand new information, the threshold falls to a minimum. In equation (iii) for the threshold, the ratio ($U_{R,a_R^*} / U_{E,a_E^*}$) located in the numerator falls. This reduces the threshold, so information has a higher chance of being evaluated emotionally by those who trade, i.e. the prevalence of noise trading rises.

Some predictions on the motivations of noise traders can be inferred: as their actions are dominated by the emotional process, the external contexts that increase the intensity of the stimuli provided by the sensory organs to the emotional process, denoted as an increase in V_E, raise the prevalence of noise trading. Therefore, data on the external context may be collected and matched to data on trades by individual traders to see if it is correlated with the dominance of the emotional process.

Also, the non-homothetic property of reward implies that an increase in the reward from the rational process will increase the threshold of the emotional process if the reward of the emotional process is kept constant, but will decrease the threshold if the ratio between both rewards is kept constant. Perhaps this property could be checked using surveys, or proxies such as GDP per capita.

3. Literature review: theory and experiments

The individual brain cell model used in this paper combines continuous spike-generation models (from neuroscience) with a discontinuous 'after-spike' reset of state variables (see (Izhikevich, 2010) for a review on hybrid spiking models of neurons). This type of model is not meant to be accurate but only sufficiently realistic and simple enough to model cell behavior.

Neuroscience and Psychology have found that memory retrieval is a slow process (see reviews in (Carrier & Pashler, 1995) and (Schall, 2003)). The brain stores prescribed guides for action (see (Bunge, 2004) and (Morris et al, 2006)), conceives new situations and remembers past experiences (see (Buckner, 2010)). The brain feels pleasure in a physical and measurable way (see (Lecknes & Tracey, 2008)). Possible rewards are played in the brain before they become realized (see (Samejima et al, 2005)). A calculation that is similar to expected utility is performed in the brain (a physical and not an *as if* calculation) (see (Plassmann et al, 2007), (Kable & Glimcher, 2007), (Knutson et al, 2005), (Symmonds et al, 2010), (Camille et al, 2011) and (Kang et al, 2011)). All of these steps take time, since evidence shows that different brain areas activate at different times (see (Bartels & Zeki, 2005)) and that time-pressure affects decision-making (see (Ordoñez & Benson, 1997)). Furthermore, experiments indicate that already chosen decisions take time to be implemented (see (Einhauser et al, 2010)).

Theoretical neuroscience is concerned with the construction of mathematical and computational models of the brain that characterize what nervous systems do and determine how they function (see (Dayan & Abbot, 2005) for an introduction). Computational neuroscience is interested in the use of computational techniques to model biological neural networks, although it also includes attempts to understand the brain and its functions through theoretical constructs (see (Arbib, 2002)).

While classical economics is largely interested in the strategies chosen by rational agents, neuroeconomics analyzes decision-making from a biological perspective (see (Camerer et al, 2005), (Vromen, 2011)). Some economists doubt the extent to which they can rely on the present corpus of neuroscience, since its predictive power is still low. More importantly, they wonder how complex behavior (such as market dynamics) could be modeled without a simple but yet general decision-making model for individuals (see (Harrison, 2008), (Kenning & Plassmann, 2005), (Keren & Schul, 2009)).

Separately, Psychology has been developing mathematical models of information transmission. An important one is the Drift Diffusion Model (DDM), proposed by Ratcliff (1978), which is based on probability theory and signal-extraction theory, and has provided a unifying framework for many experimental results in Psychology. See (Ratcliff & McKoon, 2008) for a recent review of its current use and many applications in Psychology.

In this model, an action is chosen when an accumulation of the equivalent to random utility crosses a threshold. In the case of continuous time, the DDM can be cast as a Wiener process, where the drift equals the average random utility per unit of time. However, (Proctor, 1986) observes that this model is postulated without theoretical rationale for why or when an action is chosen. This raises concerns about the generality of its predictions (for example, (Lachmann & van Leeuwen, 2010) challenges the DDM's foundations). Although our model can also be viewed as an accumulation process (see equation (13) below) it has the advantage of being based on the spike-generation models for neurons. This endows our model with a theoretical rationale capable of predicting new situations consistently and offering guidelines to harvest experimental data to calibrate behavioral models. In an unexpected direction, the model in this paper is also a generalization of the DDM: first, processes may start accumulating utility at different instants because speed is also a function of the length of the cell chain that values actions. Second, the representation step in our model incorporates the sensory system, which is not covered by the DDM, allowing modelers to take into account the effects of variations in attention on decision-making.

One branch of the neuroeconomics literature deals with conflicts between brain systems, which are treated as if each of them were rational agents that follow the expected utility model (see (Brocas & Carrillo, 2014) for a survey), and another branch incorporates some of the physiological constraints faced by the brain in the decision-making process (see (Glimcher et al, 2005) for a review).

Researchers in the fields of Psychology and Economics have found that individuals depart from the standard model of choice. (Dellavigna, 2009) consolidates this evidence. Economics has also incorporated some features that depart from the expected utility model. For example, (Brocas & Carrillo, 2012) models the process through which the brain maps evidence received from the outside world into decisions, incorporating biological constraints to the standard expected utility approach. Separately, (Bordalo et al, 2010) models choice assuming that "cognitive limitations cause people to focus their attention on some but not all aspects of the world". Similarly, (Gilboa & Schmeidler, 2002) replaces the prior used in classical expected utility with a function that selects "similar" events from memory, instead of taking into account every possible outcome. (Fudenberg & Levine, 2006) create a dual-self model and find that it gives a unified explanation of several deviations from expected utility found experimentally. (Bracha & Brown, 2012) propose a dual-self model where observed choice is the result of a pure strategy Nash equilibrium. (Clippel & Eliaz, 2012) view decision-making as an intrapersonal bargaining problem among different selves of an individual. Finally, (Harrison & Rutstrom, 2007) use data to identify whether choice is most likely generated by utility or prospect maximizers, or both, jointly estimate the parameters of each theory as well as the fraction of choices characterized by each, and find evidence that choice is generated by a mixture of utility and prospect maximizers.

4. The model

In this section, each subdivision of the model is explained. The section on Cellular Foundations explains the individual brain cells model, and the three types of cells used to understand the decision-making cycle as they have been described by neuroscience. In the section on Processes and Competition, the findings of the previous subsection are used to explain choice in terms of a competition between processes. Third, in Aggregate Behavior and Choice, we use the latter results to obtain a tractable model of decision-making similar to that of expected utility.

4.1. Cellular foundations

The brain is modelled in terms of cells and their connections. The number of cells and connections are natural numbers. The model is meant to be tractable but not fully accurate.

Cells send and receive information from other cells through connections. Each cell may receive information pulses from other cells or from sensory organs, who connect to the outside world. Each cell is filled with an inhibitory substance, which acts as a brake that prevent approaching pulses from continuing to the next cells. Although this substance stops information from continuing, the amount of this substance decreases at a rate that depends on the structure of the cell, and the amount and nature of the information received in the different information pulses that arrive. When the amount of inhibitory substance becomes sufficiently small, the cell fires a new pulse over some interval of time, towards other cells that are connected to it, by relaxing the sending cell's membrane. This pulse contains processed or modulated information. Different information may be sent towards different cells. The emission of the information pulses brings about an immediate refill of the cell with inhibitory substance, so a new cycle may start.

A cell is defined at any moment by the information it holds and the amount of inhibitory substance kept in its interior. Connections are defined by the information pulses sent from one cell to another at a given moment.

Each cell $\imath$ is a mathematical construct defined at any moment t by:

- The information contained in it, $m_\imath \epsilon \mathbb{R}^N$
- The amount of inhibitory substance, $s^\imath \epsilon \mathbb{R}^+$
- The maximum amount of inhibitory substance that the cell may contain, $\bar{s} \epsilon \mathbb{R}^+$
- The rate of decrease of inhibitory substance as it receives information, $v_\imath: \mathbb{R}^N \longrightarrow \mathbb{R}^+$
- How information is processed when sent from cell $\imath$ to cell $\jmath$, $w^{\imath\jmath}: \mathbb{R}^N \mathrm{x} \cdots \mathrm{x}\, \mathbb{R}^N \longrightarrow \mathbb{R}^N$
- The interval of time during which a cell fires information, $\xi \epsilon \mathbb{R}^+$
- The interval of time needed for information to travel from one cell to another, $\varepsilon \epsilon \mathbb{R}^+$

The state of each connection $\imath - \jmath$ at time t is defined by:

- The information sent from cell $\imath$ to cell $\jmath$,$q_{out}^{\imath\jmath} \epsilon \mathbb{R}^N$
- The information received by cell $\jmath$ from cell $\imath$, $q_{in}^{\imath\jmath} \epsilon \mathbb{R}^N$
- The amount of time needed for information sent from cell $\imath$ to cell $\jmath$, $\varepsilon \epsilon \mathbb{R}^+$
- The group of cells that share a common output information channel, $c(\imath)$, such that whenever one of them fires, all of them are immediately refilled with inhibitory substance.

To understand how cells relate to each other we need to measure how outside-world information travels this neural network model. The frequency with which cell $\imath$ throws pulses over time intervals of fixed duration ξ towards other cells is denoted as $f_\imath(t)$, and is a measure of intensity. This is what the model seeks to obtain.

The process experienced by a single cell is modelled as a spike-generation model, with the following equations:

$$(Cell)\begin{cases} \dfrac{ds_i(t)}{dt} = -\bar{s}\sum_{\hbar} v_i\big(q_{in}^{\hbar i}(t), m_i\big) + \sum_{c} \delta\big(s_{c(i)}(t) = 0\big)(\bar{s} - s_i(t)) \\ q_{out}^{ij}(t) = \begin{cases} w^{ij}\big(q_{in}^{1i}(t), q_{in}^{2i}(t), \dots, q_{in}^{Hi}(t), m_i\big) & if\ \exists\ t_0 \epsilon\ [t-\xi, t] \setminus s_i(t_0) = 0 \\ 0 & else \end{cases} \end{cases} \quad (1)$$

Where $q_{in}^{\hbar i}(t) = q_{out}^{\hbar i}(t-\varepsilon)$, describes the information pulse sent by cell $\hbar$ to cell i. The delay ε captures the fact that the information travels at a limited speed.

The rate of decrease of inhibitory substance v_i describes how much time is needed to clear s^i. When s^i becomes zero, cell i sends information q_{out}^{ij} to cell j. Finally, $\delta(s_{c(i)}(t) = 0)$ describes the instant at which the cell's membrane is relaxed and is immediately refilled with a fixed amount of inhibitory substance, called $\bar{s}$.

The following charts exemplify the evolution over time of the amount of inhibitory substance, $s^i(t)$, and of one of the possible dimensions of the information pulse $q_{out}^{ij}(t)$.

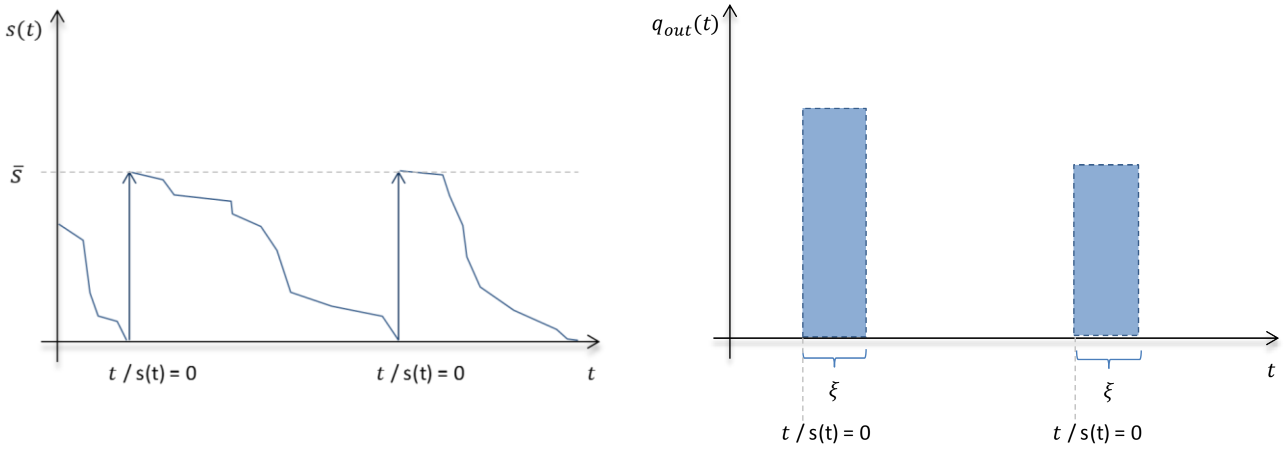


Figure 2: Evolution of the inhibitory substance and information pulses over time.

The following sections present models for each step in the choice-making cycle, all of which use the cell model presented here. The overall architecture of the model is shown below:

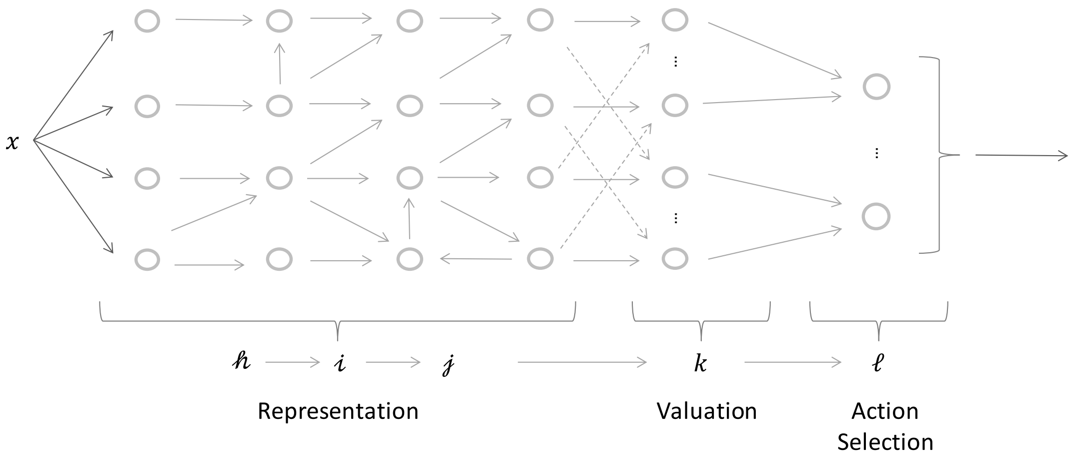


Figure 3: Steps in the decision-making cycle.

The model is designed to emulate the decision-making cycle most accepted by neuroscientists. In this model, the information about the external world that is perceived by the senses, labeled x, stimulates "representation cells". Later, the final outputs of the representation network (the current situation, potential actions, possible outcomes and memories) are fed into "valuation cells". These cells replay memories, reliving pains or pleasures, and send their assessment, together with a suggested action, to "action-selection cells". Action-selection cells "judge" suggested actions in terms of the pains and pleasures involved and send that signal on to the nervous central system, where the action is executed.

4.1.1. *Representation*

When new information is perceived, information pulses are fired towards the set of representation cells, which are, in turn, stimulated by these pulses. In particular, cell i stores memory $m_i \, \epsilon \, \mathbb{R}^N$ in a physical form and may sense outside information directly or through other representation cells.

Each representation cell is modeled by the following equations:

$$(Representation\ cell) \begin{cases} \dfrac{ds_i(t)}{dt} = -\bar{s}_i \left(\sum_{\hbar} v_i^b\big(q_{in}^{\hbar i}(t), m_i\big) + v_i^o(x_t, m_i) \right) + \bar{s}_i \; \delta(s_i(t) = 0) \\ q_{out}^{ij}(t) = \begin{cases} \sum_{\hbar} w^{ij}\big(q_{in}^{\hbar i}(t), m_i\big) & if \; \exists \; t_0 \epsilon \; [t - \xi, t] \setminus s_i(t_0) = 0 \\ 0 & else \end{cases} \end{cases} \qquad (2)$$

Where v_i^b indicates the rate at which, in representation cell i, the inhibitory substance s_i diminishes when pulses $q_{\hbar i}$ sent from brain cells $\hbar$ arrive. Separately, v_i^o indicates the rate at which the inhibitory substance s_i diminishes when outside-world information x_t arrives. Whenever s_i is reduced to zero, cell i fires a pulse of size equal to $\sum_{\hbar} w_{ij}(q_{\hbar i}(t - \varepsilon), m_i)$ towards cell j, during an interval of time that lasts ξ

Representation cells may encode many types of memories. For example, some cells may encode whole experiences and others only a few key features. Some cells may record automatic responses (in situation A, perform action B) and others may record more complex situations (in situation A, action B was performed, and consequence C occurred).

To measure the filtered impact of new information from the outside world, let us calculate $f_i(t)$, the frequency with which cell i throws pulses towards other cells. Recall that $f_i(t)$ is a measure of intensity because the time intervals have a fixed duration ξ. Theorem 1 demonstrated in appendix A, implies that:

$$f_i(t) = \sum_{\hbar} \xi \, v_i^b(w_{\hbar i}(m_{\hbar}) \, , m_i) f_{\hbar}(t - \varepsilon) + v_i^o(x_t, m_i) \tag{3}$$

The total amount of information received by a cell from other cells depends on the product of the firing rate of the feeding cell ($f_{\hbar}$) and the interval of time during which each of these latter cells fires (ξ). It also depends on v_i^b, which is the effectiveness of that information in speeding up the decrease of inhibitory substance within cell i. The intuition for (3) is that the outgoing firing rate for cell i is the product of the total amount of information it receives from feeding cells, times the “type” of that information, indicated by its ability to speed up the decrease of inhibitory substance, within cell i. The last term adds the effectiveness of the direct input from sensory organs to cell i, denoted v_i^o, in hastening the decrease of inhibitory substance inside cell i.

Matrices can be used to simplify notation. Equation (3) generalizes to:

$$f(t) = V^b \, f(t - \varepsilon) + v^o(x_t) \tag{4}$$

And thus:

$$f(t) = \sum_n (V^b)^n v^o(x_{t-n\varepsilon}) \tag{5}$$

Notice that outside-world information may influence events far into the future, depending on the form of matrix V^b. For example, if cells are interconnected in loops, outside-world information influences the behavior of representation cells for a longer time than if those same cells had been connected in layers.

Outside-world information travels through the representation system as a whole by changing the firing frequency of neurons, i.e. by changing the intensity of the information emitted by neurons. This “information wave” moves at a certain and limited speed. Consequently, if one group of cells processes certain information more slowly (if v_i^b is smaller, representing less effectiveness of that information in speeding up the decrease of inhibitory substance. reacts slowly to that set of information) then that group of cells is more likely to lose the race to submit an action. One interpretation of this result would be that complexity is time consuming, and is thus “costly”, at least for some groups of cells.

The output of the representation phase is dual: it yields both descriptions of the world (situations, either current or future), denoted σ, and pre calculated actions, a, to be implemented. Descriptions of the world or pre calculated actions may feed different cells.

4.1.2. Valuation

The contributions of valuation cells are to feel how much pain or pleasure a situation will evoke in case an action is implemented, and to fire at a frequency that reflects these cells' assessment of the likeliness of occurrence, as explained below.

At time t cell k receives information pulses from two representation cells, $\sigma(k)$ and $a(k)$. $\sigma(k)$ sends information containing a particular situation (descriptions of the world), denoted $q_{out}^{\sigma(k),k}$, while $a(k)$ sends information containing one pre calculated action $q_{out}^{a(k),k}$. When the inhibitory substance in cell k reaches zero it will fire the assessed pleasure of situation $q_{out}^{\sigma(k),k}$, which we label $u_k(q_{out}^{\sigma(k),k})$. This measure of utility is physical. The valuation cell also repeats the pre calculated action $q_{out}^{a(k),k}$, to be used in case of implementation. Both outputs are directed towards cell ℓ.

On the basis of this insight, we propose the following adaptation to the general model, where and v_k^v is the rate at which the inhibitory substance in valuation cell k diminishes:

$$(Valuation\ Cell)\begin{cases} \dfrac{ds_k(t)}{dt} = -\bar{s}_k v_k^v\left(q_{in}^{\sigma(k),k}(t), q_{in}^{a(k),k}(t)\right) + \bar{s}_k\,\delta(s_k(t)=0) \\ q_{out}^{k\ell}(t) = \begin{cases} \omega_{k\ell}\cdot\begin{bmatrix} u_k\left(q_{in}^{\sigma(k),k}(t)\right) \\ q_{in}^{a(k),k}(t)\end{bmatrix} & if\ \exists\ t_k\ \epsilon\ [t-\xi,t]\setminus s_k(t_k)=0 \\ 0 & else \end{cases}\end{cases} \tag{6}$$

The indicator variable $\omega_{k\ell}$ is defined to be equal to one if cell k sends information towards cell ℓ and zero if not. The delay ε is captured by imposing $q_{in}^{\sigma(k),k}(t) = q_{out}^{\sigma(k),k}(t-\varepsilon)$ and $q_{in}^{a(k),k}(t) = q_{out}^{a(k),k}(t-\varepsilon)$.

Now we must explain the impact of the rate at which the inhibitory substance diminishes in valuation cell k, v_k^v. This rate controls the frequency of outgoing pulses. Importantly, in the overall decision-making process, this frequency determines how much weight is awarded to the valuation $u_k\left(q_{in}^{\sigma(k),k}\right)$ and to the action $q_{out}^{a(k),k}$. As will be shown below, this frequency and the associated overall weight is similar to a probability weighting. This process means that the equivalent to the perceived probability of gaining pleasure u_k from action $q_{out}^{a(k),k}$ is captured by the rate v_k^v. This rate depends on both the situation $q_{in}^{\sigma(k),k}$ and the action $q_{out}^{a(k),k}$, allowing the cells to take into account the coherence between situation and action when setting the rate v_k^v.

The firing frequency of a valuation cell can be calculated using theorem 2, which is demonstrated in appendix B:

$$f_k(t) = \xi^2 f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) \tag{7}$$

The interval of time ξ is squared because two cells are involved.

In the next phase, $q_{in}^{a(k),k}$ will be used as a suggested action-signal and valuation $u_k\left(q_{in}^{\sigma(k),k}(t)\right)$ will be used in a calculation similar to the assignment of utils to consequences in standard economic models. Unlike those economic models, there might be different valuation functions working at the same time. Namely, $u_k\left(q_{in}^{\sigma(k),k}(t)\right)$ may change across cells k.

4.1.3. *Action selection*

In this part of the cycle, each group of cells behaves independently, and competes in letting their evaluated action-signals pass through. This is because the architecture of the network requires all these signals to pass through a single channel. After one signal passes through, every action-selection membrane will relax and cells will be refilled with inhibitory substance. Therefore, the first evaluated action that is able to pass through the action channel is the only one that will be performed by the organism.

This architectures is reflected in the following equations:

$$(Action - Selection\ Cell)\begin{cases} \dfrac{ds_\ell(t)}{dt} = -\bar{s}\displaystyle\sum_k v^{as}\left(q_{in}^{k\ell}(t)\right) + \sum_{\ell'} \delta(s_{\ell'}(t) = 0)(\bar{s} - s_\ell(t)) \\ q_{out}^{\ell}(t) = \begin{cases} w^{as}\left(q_{in}^{k\ell}(t)\right) & if\ \exists\ t_0 \epsilon\ [t-\xi, t] \setminus s_\ell(t_0) = 0 \\ 0 & else \end{cases} \end{cases} \tag{8}$$

Where, $q_{in}^{k\ell}(t) = q_{out}^{k\ell}(t-\varepsilon) = \begin{cases} \omega_{k\ell}\begin{bmatrix} q_{in}^{a(k),k}(t-\varepsilon) \\ u_k\left(q_{in}^{\sigma(k),k}(t-\varepsilon)\right) \end{bmatrix} & if\ cell\ k\ fires \\ 0 & else \end{cases}$

This version of the model also postulates that $v^{as}\left(q_{in}^{k\ell}(t)\right) = u_k\left(q_{in}^{\sigma(k),k}(t)\right)$. This assumption says that the physical measure of pain or pleasure in the valuation phase is also the rate of decrease of the inhibitory substance in the action-selection phase. It follows that actions that are evaluated as having more desirable results (higher u_k) in the valuation phase, are also processed at a higher speed in the action-selection phase.

It is also assumed that the weight $w^{as}\left(q_{in}^{k\ell}(t)\right) = q_{in}^{a(k),k}(t)$, which implies that the suggested action signal (if cell k fires) is simply copied, so this phase repeats the information created in previous phases.

We further simplify the architecture of the network by assuming that every valuation cell k that is linked to action-selection cell ℓ evaluates a unique response. Equivalently, $a(k) = a(k') = a_\ell \Leftrightarrow \omega_{k\ell} = \omega_{k'\ell} = 1$. A consequence of this simplification is that if a brain with this architecture possesses N action-selection cells it may consider a maximum of N suggested actions at the same time.

If no action cell has "fired" in a time interval of length dt, the amount of inhibitory substance in cell ℓ will have decreased by the following amount, set by theorem 3, demonstrated in appendix C:

$$ds_\ell(t) = -\bar{s}\sum_k \omega_{k\ell}\,\xi\, f_k(t-\varepsilon)u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)\,dt \tag{9}$$

Recall that here $u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)$ is the pain or pleasure felt at time $t-\varepsilon$ by cell k and $f_k(t-\varepsilon)$ is the intensity or frequency at which that cell k sent information towards cell ℓ. Let us designate the only action-taking cell that sends an order as ℓ^*. That order resets every other action-taking cell to a neutral state.

The cell whose amount of inhibitory substance reaches zero first will pass along the suggested action. Later, it will be observed as the action chosen by the organism. If cell ℓ's inhibitory substance reaches zero, it means that counting from t_0 (the last time it was filled with inhibitory substance) up to the moment it fired (labeled as t), the whole substance was absorbed:

$$-\int_{t_0}^{t} ds_\ell(t) = \bar{s} \Leftrightarrow -\int_{t_0}^{t}\frac{ds_\ell(t)}{\bar{s}} = 1 \tag{10}$$

Assume, without loss of generality, that cell ℓ^* was the one that sent the action to be performed. This means that the entire amount of inhibitory substance in cell ℓ^* was released. It also means that t^*, the time needed by cell ℓ^* to do so, was the smallest in the set of all the potential times needed by all action-selection cells to fire. Combining (9) and (10), the following two results obtain:

$$\begin{cases} -\displaystyle\int_{t_0}^{t^\ell}\frac{ds_\ell(t)}{\bar{s}} = 1 = \xi\int_{t_0}^{t^\ell}\sum_k \omega_{k\ell} f_k(t-\varepsilon)\,u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)dt \\ t^* = \min\limits_{\ell}(t^\ell) \end{cases} \tag{11a}$$

Equation (11) says that an action signal is sent by one cell if and only if it is the first among all action-selection cells to remove one hundred percent of its inhibitory substance. In this setting there is a clear difference between not reaching a decision (the inhibitory substance never clears) and choosing to do nothing (the selected action is the status quo).

4.2. *Processes and competition*

Here, the three steps are brought together to explain how decision-making can be understood in terms of a competition between processes.

During the last step, action-taking cell ℓ^* sends an order to other parts of the body to perform actions. As was shown in equation (11a), that order takes into account previous information such that:

$$\begin{cases} 1 = \xi \int_{t_0}^{t^\ell} \sum_k \omega_{k\ell} f_k(t-\varepsilon)\, u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) dt \\ t^* = \min_{\ell}\left(t^\ell\right) \end{cases} \tag{11b}$$

If cell k sends information towards cell ℓ, then frequency f_k in (11b) is given by equation (7):

$$f_k(t) = \xi^2 f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) \tag{7b}$$

Where $f_{\sigma(k)}$ and $f_{a(k)}$ are the firing frequencies of cells $\sigma(k)$ and $a(k)$.

Combining, cell ℓ sends an order when:

$$1 = \xi^3 \int_{t_0}^{t^\ell} \sum_k \omega_{k\ell} f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) dt \tag{12}$$

Next, we need to retrieve the firing frequency of representation cells $\sigma(k)$ and $a(k)$:

$$f(t) = \sum_n (V^b)^n v^o(x_{t-n\varepsilon}) \tag{5b}$$

Where f is the vector of all firing rates of representation cells, V^b is a matrix that specifies how the firing frequency of each cell changes when other cells' firing rate change, and v^o is a vector that contains how outside world information changes the firing rate of each individual cell. Thus, cells $\sigma(k)$ and $a(k)$ will send information according to that specified rate.

Combining (5b) and (12), this model of choice is completely characterized by three equations:

PROPOSITION 1: A neural model of choice can be completely characterized by three equations:

$$(Choice)\begin{cases} f(t) = \sum_n (V^b)^n v^o(x_{t-n\varepsilon}) \\ 1 = \xi^3 \int_{t_0}^{t^\ell} \sum_k \omega_{k\ell} f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) dt \\ t^* = \min_{\ell}(t^\ell) \end{cases} \tag{13}$$

Proof: For the architecture in section 4.2, using Theorems 1, 2 and 3 in appendices A, B and C.

Thus, the simplest neural processes start with perception by the senses and end in action-taking cells. These processes take time, and only the first process to complete the goal of passing a certain threshold will be executed. Since in this model actions that are evaluated as having more desirable results (higher u_k) in the valuation phase, also decrease the amount of inhibitory substance at a higher speed in the action-selection phase, these particular actions also have better chances of being selected and executed. However, this is not the only source of competitiveness for an action. Processes that start earlier (have lower t_0) and processes that receive stronger signals from the sensory organs connected to outside world (higher $f_{a(k)}\ and\ f_{\sigma(k)}$) also have better chances of being executed.

4.3. Aggregate behavior and choice

In its first part, this section identifies the special conditions required for this model's responses to become equivalent to those of a classical optimizing and forward-looking agent. Some conditions need to be imposed to fit this model of decision-making into that of expected utility.

PROPOSITION 2: The aggregate behavior of cells according to (13) is equivalent to the one predicted by expected utility theory, if and only if the following three conditions are met:

i. Information does not change during the evaluation period and travels at the same speed (every set of sensory inputs is assessed at the same time and at the same rate). This is equivalent to stating that the valuation period starts at the same time for every group of valuation cells.
ii. Every outcome must be evaluated with the same consistent criterion. This requires that either there is a single group of valuation cells, or that all groups of valuation cells have the same assessment for each and all situations and actions.
iii. Every evaluation must be exclusively based on expected outcomes.

Proof: See Appendix D.

In words, the condition that information does not change during the evaluation period and it travels at the same speed allows us to think of decision-making as if it happened between periods, just like expected utility does, without interference from sensory inputs and from changes in context that occur inside the decision-making interval. The second condition, that every outcome is evaluated with the same criterion, plays the role of a consistent utility function when evaluating memories. The third condition is the standard assumption that the agent cares about consequences only; this condition is not met in choice models derived from Prospect Theory, for example.

PROPOSITION 3: If information does not change during the evaluation period, as in condition (i) for Proposition 2, then the neural model of choice can be completely characterized by the following 2 equations:

$$(Choice)\begin{cases} V_p \equiv \sum_k w_{pk} v_k^0 \\ a^* = arg\min_{(p,a)} \quad t^{p,a} = \dfrac{N^p}{s_p} + \dfrac{1/V_p}{m_p \cdot \left(\sum_c \pi_p(a,c) \cdot u_p(a,c)\right)} \end{cases} \tag{14}$$

Proof: See Appendix E

In this setting, there are P $(p = 1,2, \dots P)$ processes that value alternative actions. P is linked to the number of action selection cells. c is the possible consequence of executing action a, $\pi_p(a,c)$ operates as the subjective probability of c given a. $u_p(a,c)$ is the feeling of pleasure process p when valuing consequence c if action a were executed.

In each process p, N^p is the number of brain cells needed to decode and evaluate information (a measure of the simplicity of the process, which may run from reflexes to slow maturation), s_p is the average speed of processing (a measure of the complexity level of the actions being evaluated), V_p is the weighted average of the intensity of the stimuli provided by the sensory organs (which are denoted v_k^0 and are weighted by the w_{pk}), and m_p reflects the overall weight that process p has earned in this organism's decision making (perhaps through experience and education).

5. Summary and conclusions

This paper models the process of decision-making in terms of neurons and their architecture. Three different types of neuronal cells and a general class of interconnection are combined. They replicate the three stages of decision-making identified in the neurology literature: representation, valuation, and action-selection.

In this neuronal model, memory retrieval is a slow process; the brain stores prescribed guides for actions; conceives new situations and remembers past experiences. The brain feels pleasure in a physical and measurable way and possible rewards are played in the brain before they become realized. A calculation that is similar to expected utility is literally performed by a specialized set of cells. In the last stage, there is a race between different groups of neurons, each of which performs the former two stages.

Analysis of this model also shows that the aggregate behavior of these brain cells can be similar to the behavior of an optimizing agent. Although no cell is actually optimizing, competition between these groups of brain cells grants the privilege of executing the organism's action to the group that finishes first..

The paper also identifies the assumptions needed for the aggregate behavior of an organism controlled by this competition between groups of neuronal cells, to be equivalent to the one predicted by standard expected utility theory.

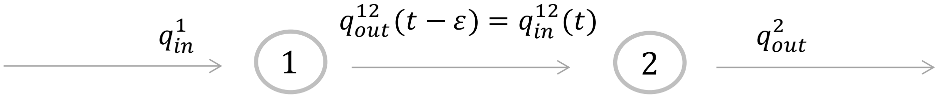
Nevertheless, the model is more general, because it allows consideration of a wider and more realistic set of situations than those that meet the assumptions required by equivalence with expected utility. For instance, these generalizations show why pure desirability (as understood in terms of utility) is not the only variable to be taken into account in choice. If a neuronal structure can use shortcuts in some situations, decisions will be biased towards them. Similarly, if the perception system changes the focus of attention towards some characteristic of alternatives, then decision-making will overweigh those processes that value these characteristics more.

For the more general case, the model yields a formula that generalizes expected utility. This decision rule yields new behavioral predictions, which should allow empirical researchers to test this model. The applications provided in section 2 help identify some of the testable implications. This model may also be embedded in market models, where several organisms participate and compete. Possible applications could be pursued by future research in the areas of intertemporal choice, financial bubbles, marketing, and behavioral economics and finance. Another direction for research is to develop this model further in the areas of self-control and learning, always based on neuroscience.

Appendices

A.-Theorem 1: The average firing rate of a cell affected by individual information pulses

In this setting, cell two receives impulses from cell one.



Cellular processes are defined by:

$$(Cell^{i})\begin{cases} \dfrac{ds^{i}(t)}{dt} = -\bar{s}\cdot v^{i}\left(q_{in}^{\hbar i}(t)\right) + \bar{s}\cdot\ \delta(s^{i}(t)=0) \\ q_{out}^{ij}(t) = \begin{cases} w^{ij}\left(q_{in}^{\hbar i}(t), m_{i}\right) & if\ \exists\ t_{0}\epsilon\ [t-\xi, t] \setminus s^{i}(t_{0}) = 0 \\ 0 & else \end{cases} \end{cases}$$

Where $q_{in}^{1,2}(t) = q_{out}^{1,2}(t-\varepsilon)$ and ξ is the firing interval as shown in figure 2.

The firing rate $f^{i}$is defined as the number of times cell i fires in a predefined interval of time, of duration ξ. According to our model, this is directly related to the number of times the inhibitory substance $s^{i}(t)$ reaches zero.Thus, an approximation for the average firing rate is the average rate of decrease of inhibitory substance, divided by the increase of inhibitory substance after new information is fired.

An approximation of the average firing rate of cell 2 in a Δt period of time is:

$$f^{2}(t, t+\Delta t) = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} -\frac{\frac{ds^{2}}{dt}(u)}{\bar{s}}\,du = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} v^{2}\left(q_{in}^{1,2}(u)\right)du$$

Now, since information pulses are received from cell 1:

$$q_{in}^{1,2}(u) = q_{out}^{1,2}(u-\varepsilon) = \begin{cases} w^{1,2}\left(q_{in}^{*,1}(u-\varepsilon)\right) & if\ \exists\ t_{0}\epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s^{1}(t_{0}) = 0 \\ 0 & else \end{cases}$$

Then,

$$f^{2}(t)\ = \frac{1}{\Delta t}\int_{t}^{t+\Delta t} v^{2}\left(q_{out}^{1,2}(u-\varepsilon)\right)du$$

$$f^{2}(t) = \frac{1}{\Delta t}\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} v^{2}\left(w^{1,2}\left(q_{in}^{*,1}(u)\right)\right)\ H(\,u\,)du$$

Where $H(\,u\,) = \begin{cases} 1 & if\ \exists\ \ t_{0}\epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s^{1}(t_{0}) =\ 0 \\ 0 & else \end{cases}$

In the special case where $q_{in}^{*,1}(u)$ is constant at the moment of firing, we find:

$$f^2(t) \; = \frac{1}{\Delta t} v^2 \left(w^{1,2} \left(q_{in}^{*,1}(t-\varepsilon, firing) \right) \right) \int_{t-\varepsilon}^{t-\varepsilon+\Delta t} H(\,u\,) du$$

The last integral can be approximated as the average firing rate of cell 1 in that period multiplied by its firing interval ξ:

$$f^2(t) = \frac{1}{\Delta t} v^2 \left(w^{1,2} \left(q_{in}^{*,1}(t-\varepsilon, firing) \right) \right) \Delta t \; \xi \; f^1(t-\varepsilon)$$

Finally, we find:

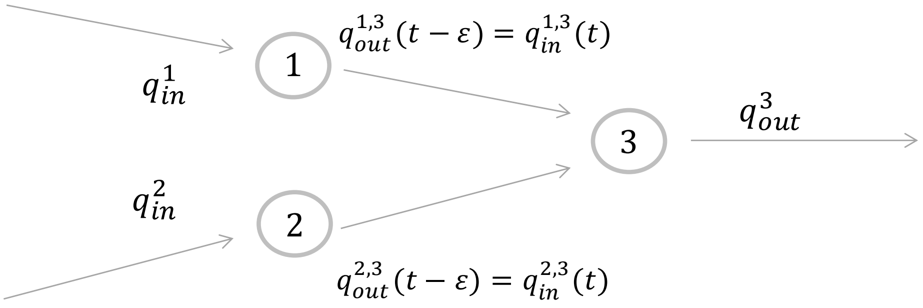
$$f^2(t) \; = \xi \; v^2 \left(w^{1,2} \left(q_{in}^{*,1}(t-\varepsilon, firing) \right) \right) f^1(t-\varepsilon)$$

$$f^2(t) \; = \xi \; v^2 \left(q_{out}^{1,2}(t-\varepsilon, firing) \right) f^1(t-\varepsilon)$$

This result is consistent with units: firing rates and v are measured in the inverse of units of time and the firing interval ξ is measured in time units.

B.-Theorem 2: The average firing rate of a cell receiving information pulses from two sources

In this setting, cell 3 receives impulses from cells 1 and 2.



Cellular processes at cell 3 are defined by:

$$\begin{cases} \dfrac{ds_3(t)}{dt} = -\bar{s} \cdot v^3 \left(q_{in}^{1,3}(t), q_{in}^{2,3}(t) \right) + \bar{s} \cdot \; \delta(s_3(t) = 0) \\ q_{out}^3(t) = \begin{cases} w^{3,*} \left(q_{in}^{1,3}(t), q_{in}^{2,3}(t) \right) & if \; \exists \; t_3 \; \epsilon \; [t-\xi, t] \setminus s_3(t_3) = 0 \\ 0 & else \end{cases} \end{cases}$$

Where $q_{in}^{1,3}(t) = q_{out}^{1,3}(t-\varepsilon)$ and $q_{in}^{2,3}(t) = q_{out}^{2,3}(t-\varepsilon)$. The important feature here is that v_3 decreases only in the intervals where both cells 1 and 2 are firing information simultaneously. Recall that each cell's firing occurs during a time interval of duration ξ.

The firing rate $f^i(t)$ is defined as the number of times cell *i* fires in a given interval. This is directly related to the number of times the inhibitory substance reaches zero. Again, an approximation of the average firing rate is the average rate of decrease of inhibitory substance, divided by the increase of inhibitory substance after new information is fired.

An approximation of the average firing rate of cell 3 in a Δt period of time is:

$$f^3(t) = \frac{1}{\Delta t}\int_t^{t+\Delta t} -\frac{\frac{ds^3}{dt}(u)}{\bar{s}}\,du = \frac{1}{\Delta t}\int_t^{t+\Delta t} v^3\left(q_{in}^{13}(u), q_{in}^{23}(u)\right)du$$

Now, since information pulses are received from cells 1 and 2:

$$q_{in}^{1,3}(u) = q_{out}^{1,3}(u-\varepsilon) = \begin{cases} w^{1,3}(q_{in}^{1}(u-\varepsilon)) & if\ \exists\ t_1 \epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s_1(t_1) = 0 \\ 0 & else \end{cases}$$

$$q_{in}^{2,3}(u) = q_{out}^{2,3}(u-\varepsilon) = \begin{cases} w^{2,3}(q_{in}^{2}(u-\varepsilon)) & if\ \exists\ t_2 \epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s_2(t_2) = 0 \\ 0 & else \end{cases}$$

The amount of inhibitory substance will decrease only if both cells send pulses. Therefore,

$$f^3(t) = \frac{1}{\Delta t}\int_t^{t+\Delta t} v^3\left(q_{out}^{1,3}(u-\varepsilon), q_{out}^{2,3}(u-\varepsilon)\right)du$$

$$f^3(t) = \frac{1}{\Delta t}\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} v^3\left(w^{1,3}\left(q_{in}^{*,1}(u-\varepsilon)\right), w^{2,3}\left(q_{in}^{*,2}(u-\varepsilon)\right)\right)H(\,u\,)du$$

Where $H(\,u\,) = \begin{cases} 1 & if\ \ \exists\ \ t_1, t_2\ \epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s_1(t_1) = s_2(t_2) = \ 0 \\ 0 & else \end{cases}$

In the special case where q_{in}^{1} and q_{in}^{2} are constant at the moment of firing, we find that:

$$f^3(t) = \frac{1}{\Delta t} v^3\left(w^{1,3}\left(q_{in}^{*,1}(t-\varepsilon, firing)\right), w^{2,3}\left(q_{in}^{*,2}(t-\varepsilon, firing)\right)\right)\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} H(\,u\,)du$$

The number of times cell 1 fires is its frequency multiplied by Δt. Thus, the time interval during which cell 1 fires is that number multiplied by the duration of its firing time. Similarly, that time multiplied by the firing frequency of cell 2 is the number of times cell 2 fires in that time interval. If we multiply that number by the firing time of cell two we find the total amount of time during which both cells are firing together. Thus, we can approximate the last integral as:

$$\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} H(\,u\,)du = \xi\ f^1(t-\varepsilon)\ \xi\ \ f^2(t-\varepsilon)\ \Delta t$$

Replacing this result, we find:

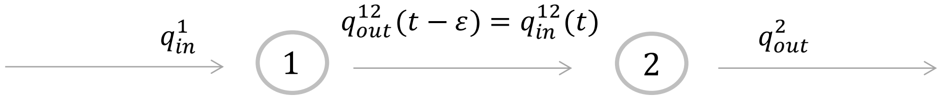
$$f^3(t) = \xi^2 v^3\left(w^{1,3}\left(q_{in}^{*,1}(t-\varepsilon, firing)\right), w^{2,3}\left(q_{in}^{*,2}(t-\varepsilon, firing)\right)\right) f^1(t-\varepsilon) f^2(t-\varepsilon)$$

$$f^3(t) = \xi^2 v^3\left(q_{out}^{1,3}(t-\varepsilon, firing), q_{out}^{2,3}(t-\varepsilon, firing)\right) f^1(t-\varepsilon) f^2(t-\varepsilon)$$

This result is consistent with units: firing rates and v are measured in the inverse of units of time and ξ in time units.

C.-Theorem 3: The average decrease of inhibitory substance in one cell

We will show that the average decrease of inhibitory substance in a cell that does not fire an information pulse is related to the average firing rate of the previous cell. In this setting, cell two receives impulses from cell one.



Cellular processes are defined by:

$$(Cell^{i})\begin{cases} \dfrac{ds^{i}(t)}{dt} = -\bar{s}\cdot v^{i}\left(q_{in}^{\hbar i}(t)\right) + \bar{s}\cdot\ \delta(s^{i}(t)=0) \\ q_{out}^{ij}(t) = \begin{cases} w^{ij}\left(q_{in}^{\hbar i}(t), m_{i}\right) & if\ \exists\ t_{0}\epsilon\ [t-\xi, t] \setminus s^{i}(t_{0}) = 0 \\ 0 & else \end{cases} \end{cases}$$

Where $q_{in}^{1,2}(t) = q_{out}^{1,2}(t-\varepsilon)$.

From the first equation, we know that, if cell 2 has not fired, its average decrease in inhibitory substance is given by:

$$d\bar{s}^{2}(t) = -\bar{s}\frac{1}{\Delta t}\int_{t}^{t+\Delta t} v^{2}\left(q_{in}^{1,2}(u)\right) du\, dt$$

Now, since information pulses are received from cell 1:

$$q_{in}^{1,2}(u) = q_{out}^{1,2}(u-\varepsilon) = \begin{cases} w^{1,2}\left(q_{in}^{*,1}(u-\varepsilon)\right) & if\ \exists\ t_{0}\epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s^{1}(t_{0}) = 0 \\ 0 & else \end{cases}$$

Where $q_{in}^{*,1}$ is the information that cell 1 receives from the previous layer of cells.

Then,

$$d\bar{s}^{2}(t) = -\bar{s}\frac{1}{\Delta t}\int_{t}^{t+\Delta t} v^{2}\left(w^{1,2}\left(q_{in}^{*,1}(u-\varepsilon)\right)\right)\ du\, dt$$

$$d\bar{s}^{2}(t) =\ -\bar{s}\frac{1}{\Delta T}\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} v^{2}\left(w^{1,2}\left(q_{in}^{*,1}(u)\right)\right) H(\,u\,) du\, dt$$

Where $H(\,u\,) = \begin{cases} 1 & if\ \exists\quad t_{0}\epsilon\ [u-\varepsilon-\xi, u-\varepsilon] \setminus s^{1}(t_{0}) =\ 0 \\ 0 & else \end{cases}$

In the special case where $q_{in}^{*,1}$ is constant at the moment of firing, we find:

$$d\bar{s}^2(t) = -\bar{s}\frac{1}{\Delta t}v^2\left(w^{1,2}\left(q_{in}^{*,1}(t-\varepsilon, firing)\right)\right)\int_{t-\varepsilon}^{t-\varepsilon+\Delta t} H(\,u\,)du\,dt$$

The last integral can be approximated as the average firing rate of cell 1 in that period multiplied by its firing length:

$$d\bar{s}^2(t) = -\bar{s}\frac{1}{\Delta t}v^2\left(w^{1,2}\left(q_{in}^{*,1}(t-\varepsilon, firing)\right)\right)\Delta t\,\xi\, f^1(t-\varepsilon)dt$$

Finally, we find:

$$d\bar{s}^2(t) = -\bar{s}\,\xi\, v^2\left(w^{1,2}\left(q_{in}^{*,1}(t-\varepsilon, firing)\right)\right)f^1(t-\varepsilon)dt$$

$$d\bar{s}^2(t) = -\bar{s}\,\xi\, v^2\left(q_{out}^{1,2}(t-\varepsilon, firing)\right)f^1(t-\varepsilon)dt$$

This is consistent with units: $d\bar{s}^2$ and $\bar{s}$ are measured in amount of inhibitory substance, firing rates and v are measured in the inverse of units of time and ξ and dt are measured in time units.

D.-PROPOSITION 2: Case of equivalency with classical Expected Utility

This appendix shows that this model of choice based on cell behavior can be interpreted as a maximization of expected utility if the following conditions are met:

1. Information does not vary over the period of evaluation and every representation influences action-selection cells through valuation cells at the same moment. This means that matrix V^b is such that representation cells that send information to valuation cells do so at the same moment (t_0 is constant for every valuation cell) and the information flow is constant over time.
2. Every outcome must be evaluated with the same and consistent criterion (experienced utility based on preferences). This means $u_k(.) = u(.)$ for all cells k.
3. Every evaluation is exclusively based on expected outcomes. This means that $q_{out}^{\sigma(k),k}$ is always a consequence and never a previous experience.

Part I: If the above conditions are met, then the model is equivalent to a utility maximizing agent.

The equation that describes the moment at which cell ℓ fires is given by:

$$1 = \xi\int_{t_0}^{t^\ell}\sum_k \omega_{k\ell}f_k(t-\varepsilon)u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)dt$$

Condition 1 means that the rate at which inhibitory substance decreases is constant. Thus:

$$\int_{t_0}^{t^\ell}\sum_k \omega_{k\ell}f_k(t-\varepsilon)u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)dt = \left(t^\ell - t_0\right)\sum_k \omega_{k\ell}f_k u_k\left(q_{out}^{\sigma(k),k}\right)$$

Where $t^{\ell} \in [t_0, t_0 + \xi]$ is the interval during which cell ℓ would have fired a signal describing a suggested action, if it were the first action-selection cell to do so. Then,

$$t^{\ell} = t_0 + \frac{1}{\xi} \frac{1}{\sum_k \omega_{k\ell} f_k u_k (q_{out}^{\sigma(k),k})}$$

It follows that, if $t^* = \min_{\ell}(t^{\ell})$, then $\left(q_{out}^{a(k),k}\right)^*$, the performed suggested action, must maximize the denominator, so:

$$\left(q_{out}^{a_{\ell},k}\right)^* = \underset{m_{a_{\ell}}}{\operatorname{argmax}} \sum_k \omega_{k\ell} f_k u_k (q_{out}^{\sigma(k),k})$$

We can expand and re-write this last equation as:

$$m_{a_{\ell}}{}^* = \underset{m_{a_{\ell}}}{\operatorname{argmax}} \sum_k \omega_{k\ell} f_{\sigma(k)} f_{a_{\ell}} v_k^v\left(q_{out}^{\sigma(k),k}, q_{out}^{a_{\ell},k}\right) u_k (q_{out}^{\sigma(k),k})$$

Besides, if $u_k(q_{out}^{\sigma(k),k}) = u(q_{out}^{\sigma(k),k})$ (condition 2), and $q_{out}^{\sigma(k),k}$is always a consequence (condition 3), this formula becomes the equivalent of a linear transformation of an expected utility.

In particular, the functional form of the subjective probability measure used to value expectations is of the form:

$$P(q_{out}^{\sigma(k),k} / q_{out}^{a(k),k}) = \frac{\omega_{k\ell} f_{\sigma(k)} f_{a(k)} v_k^v\left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k}\right)}{\sum_k \omega_{k\ell} f_{\sigma(k)} f_{a(k)} v_k^v\left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k}\right)}$$

Part II: If the above conditions are met, then a utility maximizing agent acts equivalently to a decision-maker that processes information as modelled in this paper.

The model shows actions $q_{out}^{a(k),k}$ can be ordered as in preferences in terms of t^{ℓ}, the time they need to clear the inhibitory substance in action-selection cell ℓ (remember there is only one action per action-selection cell). Then:$q_{out}^{a(k),k} \succcurlyeq q_{out}^{a(k)\prime,k} \Leftrightarrow t^{\ell} \leq t^{\ell'}$.

Besides, t^{ℓ} is defined by the formula:

$$1 = \xi^3 \int_{t_0}^{t^{\ell}} \sum_k \omega_{k\ell} f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t - \varepsilon) v_k^v \left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) u_k \left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) dt$$

The last formula can be re-written as follows:

$$\frac{1}{\xi^3} = \left(t^\ell - t_0\right)\left[\sum_{k/q_{out}^{\sigma(k),k} \in C} \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\bar{u}\left(\bar{q}_{out}^{\sigma(k),k}\right)\right] \tag{a}$$

$$+\left(t^\ell - t_0\right)\left[\sum_{k} \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\bar{u}\left(\bar{q}_{out}^{\sigma(k),k}\right) - \sum_{k/q_{out}^{\sigma(k),k} \in C} \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\bar{u}\left(\bar{q}_{out}^{\sigma(k),k}\right)\right] \tag{b}$$

$$+\left(t^\ell - t_0\right)\left[\sum_{k} \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\left(u_k\left(\bar{q}_{out}^{\sigma(k),k}\right) - \bar{u}\left(\bar{q}_{out}^{\sigma(k),k}\right)\right)\right] \tag{c}$$

$$+\left[\sum_{k}\int_{t_0}^{t^\ell} \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\left(u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) - u_k\left(\bar{q}_{out}^{\sigma(k),k}\right)\right)dt\right] \tag{d}$$

$$+\left[\sum_{k}\int_{t_0}^{t^\ell} u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)\left(g_{a(k)}(t, q_{out}^{\sigma(k),k}) - \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right)\right)dt\right] \tag{e}$$

Where

$$\begin{cases} g_{a(k)}(t, q_{out}^{\sigma(k),k}) = \omega_{k\ell} f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) \\ \bar{g}_{a(k)}\left(q_{out}^{\sigma(k),k}\right) = \dfrac{1}{(t^\ell - t_0)}\displaystyle\int_{t_0}^{t^\ell} g_{a(k)}(t, q_{out}^{\sigma(k),k})dt \\ u_k\left(\bar{q}_{out}^{\sigma(k),k}\right) = \dfrac{1}{(t^\ell - t_0)}\displaystyle\int_{t_0}^{t^\ell} u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right)dt \\ \bar{u}\left(\bar{q}_{out}^{\sigma(k),k}\right) = \dfrac{1}{K}\displaystyle\sum_k u_k\left(\bar{q}_{out}^{\sigma(k),k}\right) \\ C \text{ is the set of situations that consists only of consequences} \\ \text{and } K \text{ is the total number of valuation cells} \end{cases}$$

Thus, t^ℓ will become a transformation of an expected utility function only if the last four terms in the equation are always equal to zero. Terms e) and d) represent condition 1, term c) represents condition 2, and term b) represents condition 3. QED

E.-PROPOSITION 3: Constrained neural model of choice

This appendix shows that it is possible to derive a tractable model of decision-making -similar to that of expected utility- that encompasses a competition between neural processes, including automatic responses and attentional biases.

First, we use equation (13) as a model of choice based on neurons:

$$(Choice)\begin{cases} f(t) = \sum_n (V^b)^n v^o(x_{t-n\varepsilon}) \\ 1 = \xi^3 \int_{t_0}^{t^\ell} \sum_k \omega_{k\ell} f_{\sigma(k)}(t-\varepsilon) f_{a(k)}(t-\varepsilon) v_k^v\left(q_{out}^{\sigma(k),k}(t-\varepsilon), q_{out}^{a(k),k}(t-\varepsilon)\right) u_k\left(q_{out}^{\sigma(k),k}(t-\varepsilon)\right) dt \\ t^* = \min_\ell (t^\ell) \end{cases}$$

If we assume that the information flow is constant over time (as in a steady state), we can easily integrate over time in the equation in the middle, to find that:

$$(Choice)\begin{cases} f = (I - V^b)^{-1} v^o \\ t^* = \min_\ell \quad t^\ell = t_0^\ell + \dfrac{1/\left(\xi^3 f_{a(k)}\right)}{\sum_k \omega_{k\ell} f_{\sigma(k)} v_k^v \left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k}\right) u_k\left(q_{out}^{\sigma(k),k}\right)} \end{cases}$$

In words, under this condition, the total time it takes to clear all the inhibitory substance can be calculated as the time needed for information to arrive plus the time to clear the substance. Recall that t_0^ℓ is the time at which information reaches the action-taking cell ℓ. This can be calculated as the ratio of the number of cells active in the process to the average speed of decoding (given by v_i^b in equation (2)).

Finally, we can re-arrange the previous equation to find the executed action a^* as the one that takes the least time $t^{p,a}$ to process action a through process p:

$$(Choice)\begin{cases} V_p \equiv \sum_k w_{pk} v_k^0 \\ a^* = arg\min_{(p,a)} \quad t^{p,a} = \dfrac{N^p}{s_p} + \dfrac{1/V_p}{m_p \cdot \left(\sum_c \pi_p(a,c) \cdot u_p(a,c)\right)} \end{cases}$$

Where the new variables are defined as follows:

$$u_p(a,c) \equiv u_k\left(q_{out}^{\sigma(k),k}\right)$$

$$\pi_p(a,c) \equiv \omega_{k\ell} f_{\sigma(k)} v_k^v \left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k}\right) / \left(\sum_k \omega_{k\ell} f_{\sigma(k)} v_k^v \left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k}\right)\right)$$

$$m_p \equiv \xi^3 \sum_k \omega_{k\ell} f_{\sigma(k)} v_k^v \left(q_{out}^{\sigma(k),k}, q_{out}^{a(k),k} \right)$$

$$f_{a(k)} \equiv f_p$$

References

Arbib, M. The Handbook Of Brain Theory And Neural Networks, Second Edition, MIT Press, 2002.

Barberis, N. & Thaler, R. «A survery of behavioral finance,» de Handbook of the Economics of Finance: Financial Markets and Asset Pricing, Elsevier Science BV, 2003, pp. 1051 - 1121 .

Bartels, A. & Zeki, S. «The chronoarchitecture of the cerebral cortex,» Philosophical Transactions of the Royal Society of London, vol. 360, nº B, pp. 733-750, 2005.

Bordalo, P., Gennaioli, N. & Shleifer, A. «Salience Theory of Choice under Risk,» NBER working paper 16387, 2010.

Bracha A., Brown, D.J., «Affective decision making: A theory of optimism bias. », Games and Economic Behavior vol 75 n 1, pp 67-80, 2012.

Brocas, I. & Carrillo, J. «From perception to action: An economic model of brain processes,» Games and Economic Behavior, vol. 75, nº 1, pp. 81-103, 2012.

Brocas, I., & Carrillo, J. «Dual-process theories of decision-making: a selective survey» Journal of Economic Psychology, 2014.

Buckner, R. «The role of Hippocampus in prediction and imagination,» Annual review of psychology, vol. 61, pp. 27-48, 2010.

Bunge, S. A. «How we use rules to select actions: A review of evidence from cognitive neuroscience,» Cognitive, Affective, & Behavioral Neuroscience, vol. 4, nº 4, pp. 564-579, 2004.

Camerer, C., Loewenstein, G. & Prelec, D. «Neuroeconomics: How Neuroscience Can Inform Economics,» Journal of Economic Literature, vol. 43, pp. 9-64, 2005.

Camille, N., Griffiths, C., Vo, K., Fellows, L. & Kable, J. «Ventromedial Frontal Lobe Damage Disrupts Value Maximization in Humans,» The Journal of Neuroscience, vol. 31, nº 20, pp. 7527-7532, 2011.

Carrier L., & Pashler, H. «Attentional Limits in Memory Retrieval,» Journal of Experimental Psychology: Learning, Memory and Cognition, vol. 21, nº 5, pp. 1339-1348, 1995.

Dayan, P. & Abbott, L. Theoretical Neuroscience. Computational and Mathematical Modeling of Neural Systems, MIT Press, 2005.

DellaVigna S., «Psychology and Economics: Evidence from the Field», Journal of Economic Literature, vol. 47, n 2, pp. 315–372, 2009.

De Clippel, G., Eliaz, K. «Reason-based choice: A bargaining rationale for the attraction and compromise effects. », Theoretical Economics vol 7, n 1, pp. 125-162, 2012.

Einhauser, W., Koch, C. & Carter, O. «Pupil dilation betrays the timing of decisions,» Frontiers in human neuroscience, vol. 4, nº 18, pp. 1-9, 2010.

Fudenberg, D., Levine, D.K., «A Dual-Self Model of Impulse Control», The American Economic Review, vol. 96, n 5, 1449-1476, 2006.

Gilboa, I. & Schmeidler, D. «Utility in case-based decision theory,» Journal of Economic Theory, vol. 105, nº 2, pp. 483-502, 2002.

Glimcher, P., Dorris, M. & Bayer, H. «Physiological utility theory and the neuroeconomics of choice,» Games and Economic Behavior, vol. 52, nº 2, pp. 213-256, 2005.

Harrison, G. «Neuroeconomics: A Critical Reconsideration,» Economics and Philosophy, vol. 24, nº 3, pp. 303-344, 2008.

Harrison, G.W., Rutström E.E., «Expected utility theory and prospect theory: one wedding and a decent funeral.» Experimental Economics vol. 12, n 2, pp 133-158, 2009.

Izhikevich, E. M. «Hybrid spiking models,» Philosophical Transactions of the Royal Society, vol. 368, p. 5061–5070, 2010.

Kable, J. & Glimcher, P. «The neural correlates of subjective value during intertemporal choice,» Nature Neuroscience, vol. 10, pp. 1625-1633, 2007.

Kang, M., Rangel, A., Camus, M. & Camerer, C. «Hypothetical and Real Choice Differentially Activate Common Valuation Areas,» The Journal of Neuroscience, vol. 31, nº 2, pp. 461-468, 2011.

Kenning, P. & Plassmann, H. «NeuroEconomics: An overview from an economic perspective,» Brain Research Bulletin, vol. 67, pp. 343-354, 2005.

Keren, G., Schul, Y.. «Two Is Not Always Better Than One: A Critical Evaluation of Two-System Theories.» Perspectives on Psychological Science, vol 4, n 6, pp 533-550, 2009.

Knutson, B., Taylor, J., Kaufman, M. Peterson, R. & Glover, G. «Distributed Neural Representation of Expected Value,» The Journal of Neuroscience, vol. 25, nº 19, pp. 4806-4812, 2005.

Lachmann, T. & van Leeuwen, C. «Representational economy, not processing speed, determines preferred processing strategy of visual patterns,» Acta Psychologica, vol. 134, nº 3, pp. 290-298, 2010.

Leknes S., & Tracey, I. «A common neurobiology for pain and pleasure,» Nature reviews, vol. 9, pp. 314-320, 2008.

Morris, G., Nevet, A., Arkadir, D., Vaadia E., & H. Bergman, «Midbrain dopamine neurons encode decisions for future action,» Nature Neuroscience, vol. 9, pp. 1057-1063, 2006.

Ordoñez, L. & Benson, L. «Decisions under Time Pressure: How Time Constraint Affects Risky Decision Making,» Organizational Behavior and Human Decision Processes, vol. 71, nº 2, pp. 121-140, 1997.

Plassmann, H., O'Doherty, J. & Rangel, A. «Orbitofrontal Cortex Encodes Willingness to Pay in Everyday Economic Transactions,» The Journal of Neuroscience, vol. 27, nº 37, p. 9984 –9988, 2007.

Proctor, R. «Response bias, criteria settings, and the fast-same phenomenon: A reply to Ratcliff,» Psychological Review, vol. 93, nº 4, pp. 473-477, 1986.

Rangel, A., Camerer C., & Montague, P. «A framework for studying the neurobiology of value-based decision making,» Nature reviews, Neuroscience, vol. 9, pp. 1-13, 2008.

Ratcliff, R. & McKoon, G. «The Diffusion Decision Model: Theory and Data for Two-Choice Decision Tasks,» Neural Computation, nº 20, pp. 874 - 922, 2008.

Saavedra, J. «It's all in the Brain,» CESifo Newsletter, nº 4424, 2013.

Samejima, K., Ueda, Y., Doya K. & Kimura, M. «Representation of action-specific reward values in the striatum,» Science, nº 310, p. 1337–1340, 2005.

Schall, J. «Neural Correlates of Decision Processses: Neural and Mental Chronometry,» Current Opinion in Neurobiology, vol. 13, nº 2, pp. 182-186, 2003.

Symmonds, M., Bossaerts, P. & Dolan, R. «A behavioral and Neural Evaluation of Prospective Decision-Making under Risk,» The Journal of Neuroscience, vol. 30, nº 43, pp. 14380-14389, 2010.

Valdes-Edwards, G. & Valdes-Prieto, S. «A tractable theory of choice based on cell behavior,» CESifo working paper, nº 4424, 2014.

Vromen, J. «Neuroeconomics: two camps gradually converging: what can economics gain from it?,» International Review of Economics, vol. 58, nº 3, pp. 267-285, 2011.