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2020

Bridge Proxy-SVAR: Estimating the Macroeconomic Effects of Shocks Identified
at High-Frequency

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Bridge Proxy-SVAR: Estimating the Macroeconomic Effects of Shocks Identified at High-Frequency*

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January 30, 2020

Abstract

This paper proposes a novel methodology, the *Bridge Proxy-SVAR*, which exploits high-frequency information for the identification of the Vector Autoregressive (VAR) models employed in macroeconomic analysis. The methodology is comprised of three steps: (I) identifying the structural shocks of interest in high-frequency systems; (II) aggregating the series of high-frequency shocks at a lower frequency; and (III) using the aggregated series of shocks as a proxy for the corresponding structural shock in lower frequency VARs. We show that the methodology correctly recovers the impact effect of the shocks, both formally and in Monte Carlo experiments. Thus the *Bridge Proxy-SVAR* can improve causal inference in macroeconomics that typically relies on VARs identified at low-frequency. In an empirical application, we identify uncertainty shocks in the U.S. by imposing weaker restrictions relative to the existing literature and find that they induce mildly recessionary effects.

JEL classification: C32; C36; E32

Keywords: structural vector autoregression; external instrument; high-frequency identification; proxy variable; uncertainty shocks.

*First Version: May 24th, 2016. We are grateful to Evi Pappa, Fabio Canova and Juan Dolado for fruitful discussions and suggestions. We also thank Ambrogio Cesa-Bianchi, Paul Beaudry, Danilo Cascaldi, Maarten Dossche, Luca Gambetti, Aeimit Lakdawala, Peter Hansen, Matteo Iacoviello, Francesca Loria, Riccardo Jack Luchetti, Kurt Lunsford, Michele Piffer, Morten Ravn, Juan Rubio-Ramirez, Andreas Tryphonides, Srećko Zimic, and seminar participants at the Bank of Italy, CEPR-EABCN-UPF Conference on “Measuring the Effects of Unconventional Monetary Policy: What Have We Learned?”, Computational Financial Econometrics Workshop, EEA-ESEM 2018, European Central Bank, European University Institute, IAAE 2018, LACEA-LAMES 2018, 4th Macro Banking and Finance Workshop, Santiago Macroeconomics Workshop, SED 2019 Annual Meeting, Side Workshop in Econometrics, Universidad Carlos III de Madrid, Universidad de Chile, Universidad Diego Portales, Universidad de San Andrés, SECHI 2019, and the CSEF for helpful comments and suggestions. A previous version of this paper which circulated with the title: “Proxy-SVAR as a Bridge between Mixed Frequencies” was awarded the *Unicredit “Macro, Banking and Finance” Best Paper Award 2016*. The views expressed in the paper are those of the authors only and do not involve the responsibility of the Bank of Italy.

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1 Introduction

The identification of causal relationships in macroeconomics is a challenging task because the exogenous innovations are mixed with the endogenous dynamics of the variables and multiple shocks affect the economy simultaneously. A recent strand of literature exploits fluctuations occurring in a narrow time window around specific events to identify a particular structural shock of interest. This identification strategy implicitly assumes that only this shock affects the economy during this short time span, which is arguably a milder restriction than those traditionally imposed in monthly or quarterly Vector Autoregressive models (VARs). To compute the effects of the innovations on macroeconomic variables, the shocks identified at the daily/intra-daily frequency are aggregated to a monthly/quarterly frequency, usually as the simple average or moving average, and used as a proxy for the (unobserved) structural shock in VAR models. In an influential work, [Gertler and Karadi \(2015\)](#) identify the macroeconomic effects of monetary policy shocks by using the series of monetary policy surprises, defined by [Gurkaynak et al. \(2005\)](#) as the change in the price of Fed Fund futures in the day of FOMC meetings, aggregated at the monthly frequency. Pioneered in the monetary policy literature, this approach has spread to other fields of research.¹

This “event-study high-frequency identification” faces two main limitations. First, it may confound the shock of interest with other shocks that occur during the same temporal window. For example, monetary policy surprises intertwine the actual monetary policy shock with releases of information on the state of the economy (see, for example, [Ramey, 2016](#) and [Miranda Agrippino and Ricco, 2018](#)). Second, several

¹Works that use a similar strategy are, among others, [Piffer and Podstawski \(2018\)](#) who use the variation in the price of gold on specific days to identify innovations in uncertainty and [Bahaj \(2019\)](#) who follows a similar approach to investigate the macroeconomic effects of changes in sovereign spreads. A related approach is taken in [Kanzig \(2019\)](#) who gauges oil supply innovations from financial markets in response to OPEC meetings.

empirical applications, typically those that do not study the effect of economic policies, cannot rely on specific events. For instance, financial shocks, understood as exogenous shifts in credit spreads (Gilchrist and Zakrajsek, 2012), can hardly be isolated through event-studies. Both issues limit the implementability of high-frequency identification.

To overcome these limitations, this paper proposes a novel methodology, the “*Bridge Proxy-SVAR*” (Bridge-PSVAR). This methodology allows the high-frequency (HF) identification of structural shocks in a general framework, deals explicitly with the correct aggregation of HF shocks to lower frequencies, and thus allows us to correctly recover the macroeconomic effects of the HF shocks. The Bridge-PSVAR is comprised of three steps.

First, we identify the structural shocks of interest in a high-frequency (e.g. daily) VAR (HF-VAR), which enables us to timely control for the information set that agents use when making decisions. In other words, the HF-VAR is subject to less severe identification challenges because it is less (or not) affected by the temporal aggregation bias.² The identification strategies typically employed in monthly or quarterly VARs (for example the recursive, sign, narrative restrictions, along with many others) can be

²The temporal aggregation bias is described in Sims (1971), Christiano and Eichenbaum (1987), Marcellino (1999), Hendry (1992), and Swanson and Granger (1997), among others. To avoid this bias, the literature has proposed using mixed frequency VAR models that handle data sampled at different frequencies, which are typically applied for forecasting. There are two main approaches to estimate VARs with mixed frequency data. The most popular one, developed by Zadrozny (1988), is based on a state space representation. While the Kalman filter has been shown to be the optimal filter in this framework, the system is driven by latent shocks whose economic interpretations are not straightforward. Moreover, the computational intensity of this technique increases exponentially with the number of states and the frequency mismatch (in particular in case of irregular frequencies as in daily-monthly or daily-quarterly). The second approach, proposed by Ghysels (2016), is more similar to standard VARs in being driven only by shocks to observable variables. This particular type of VAR deals with series sampled at different frequencies through stacking; a variable sampled at higher frequency is decomposed into several lower frequency variables and directly employed in the VAR. Its shortcoming consists of the curse of dimensionality, i.e. parameter proliferation. Due to shortcomings of the mixed frequency VAR in dealing with large frequency mismatches and since our focus is structural analysis, we take a very different approach that exploits two VAR systems specified at different frequencies. Directly related to our work is the severity of temporal aggregation biases in SVAR models as illustrated in Marcellino (1999) and Foroni and Marcellino (2016): impulse response functions and the forecast error variance decomposition can be strongly biased by temporal aggregation.

exploited to identify the shock of interest at high-frequency.³ Notably, those identification restrictions impose milder constraints on the data at higher frequencies. For instance, our empirical application exemplifies that imposing the same timing restrictions on daily versus monthly data may significantly affect the conclusions of the analysis.

Second, we compute the average of the identified (high-frequency) series of shocks at the lower frequency. We show that averaging yields the correct low-frequency structural shock due to the linearity of VAR models. Further, we test that the aggregated series of shocks are orthogonal to past information (see [Forni and Gambetti, 2014](#); [Stock and Watson, 2018](#); [Miranda Agrippino and Ricco, 2018](#); among others), which means that the HF-VAR is not information deficient.

Third, we use the aggregated series of shocks as a proxy for the corresponding structural shock at the lower frequency; from now on referred to as LF (see [Stock and Watson, 2012](#) and [Mertens and Ravn, 2013](#)).⁴ Namely, we draw identifying restrictions for the LF representation from HF information. The economic intuition is that the HF identification does not confound the actual shocks with the endogenous responses of the system to the shocks.

By blending the VAR identification techniques with HF data, until now exploited only in event-studies, our methodology: *i*) exploits all the available observations at HF and does not require pinpointing special events, which may not convey enough information for reliable statistical inference; *ii*) employs the correct, timely updated, information set of agents (e.g. it varies day by day) and thus does not confound the actual shocks with their endogenous responses;⁵ and *iii*) ensures that the shock of interest is not contaminated by

³The resulting structural shocks are separated from other economic disturbances under a valid identification strategy. As we mentioned above, this is not always the case in event-studies.

⁴While Impulse Response Function (IRFs) and other relevant statistics can be also computed in alternative ways (for example using local projections), VARs identified with the external or internal instrument constitute our benchmark. See Section 3 for a more detailed discussion on this point.

⁵Formally, this guarantees that the reduced-form residuals (and consequently the structural shocks) of

other ones, conditional on the identification strategy applied.

This paper provides a flexible tool that achieves identification of VARs based on HF data, opposed to monthly or quarterly data. The validity of the Bridge-PSVAR rests upon both analytical econometric results and on Monte Carlo simulations. First, we show formally that, if the underlying process is a VAR, the causal effects of the shocks can be recovered by aggregating the HF shocks as averages over the LF period and projecting the endogenous variables of interest on the aggregated shock. The underlying intuition stems from the linearity of VAR models and the corresponding IRFs: both the impact and dynamic effects of the shocks on the endogenous variables are linear functions of the shocks. Second, we rely on Monte Carlo experiments to provide a general assessment of the small-sample performances of the Bridge-PSVAR and to quantify the gains compared to the common practice of LF identification. In particular, we compare our procedure to a VAR identified using temporally aggregated data (LF-VAR) and to the best possible (counter-factual) HF estimation (HF-VAR). Our results show that the Bridge-PSVAR is a suitable method for approximating the true underlying responses under different data generating processes. It correctly recovers the impact effect of the shocks at lower frequencies. In fact, the Bridge-PSVAR greatly outperforms the LF-VAR, with a reduction in the temporal aggregation bias between 19% to 85% depending on the specification of the experimental setup, and yields similar but less precise estimates than the (counterfactual) HF-VAR.

The potential of the methodology is highlighted through an empirical application on uncertainty shocks. We revise the analysis of [Berger et al. \(2019\)](#) (BDG) who study the macroeconomic effects of uncertainty and realized volatility shocks in the equity market by employing a monthly VAR model of the US economy. BDG identify uncertainty shocks as the innovations that maximize the forecast error variance of expected volatility

the VAR system are defined as innovations with respect to the information set of agents.

(proxied by the VIX) over 24 months, but that are orthogonal to contemporaneous changes in realized volatility (proxied by the monthly average of daily squared returns). They show that, after controlling for realized volatility, uncertainty shocks do not induce significant macroeconomic effects in contrast with previous works (see, for example, [Bloom, 2009](#); [Basu and Bundick, 2017](#)). Realized volatility shocks instead induce sizable recessionary effects. However, assuming that realized volatility cannot respond to uncertainty shocks for a whole month may be regarded as a strong identification restriction since financial variables react immediately to new information, and this constraint may affect BDG conclusions on the role of uncertainty shocks. We apply the strategy proposed by BDG but shift the identification stage to a daily VAR, restricting to zero the response of realized volatility to uncertainty shocks within the same day instead of within the same month. Then the Bridge Proxy-SVAR is employed to identify the macroeconomic effects of exogenous fluctuations in uncertainty. We find that, contrary to the BDG monthly zero restriction, uncertainty shocks induce a significant response of realized volatility within the same month and affect employment and industrial production in a qualitatively similar manner to realized volatility shocks.

The remainder of this paper is organized as follows. Section [2](#) describes an analytical example to illustrate the econometric problem. Section [3](#) illustrates the Bridge-PSVAR methodology and its analytical properties. Section [4](#) presents the Monte Carlo experiments employed for testing the methodology. Section [5](#) studies the dynamic effects of uncertainty shocks. Finally, Section [6](#) concludes.

2 An Illustrative Case

This section describes a simple analytical case to illustrate the econometric problem, i.e. structural identification under temporal aggregation, and our proposed solution, i.e. the

Bridge-PSVAR that maintains the identification at HF and correctly aggregates the HF shocks to the LF. We postpone the full characterization of the econometric framework to Section 3.

Consider the simple (covariance stationary and causal) bivariate $VAR(1)$ process at the HF frequency $t = 1, 2, \dots, T$ in Eq.(1):

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^y \end{bmatrix} \quad (1)$$

where $[x_t \ y_t]'$ are the two endogenous variables, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is the autoregressive matrix, $B = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix}$ is the impact matrix that determines the instantaneous feedback from the structural innovations $\varepsilon_t = [\varepsilon_t^x \ \varepsilon_t^y]'$ to the endogenous variables, where $\mathbb{E} [\varepsilon_t \varepsilon_t'] = \mathcal{I}_2$. For simplicity but without loss of generality, we assume B to be lower triangular, which means that ε_t^y does not affect x_t within the same period. However, from the estimation of the reduced form system only a linear combination of them is available to the econometrician. The corresponding reduced form residuals $u_t = B\varepsilon_t$ are indeed correlated: $\mathbb{E} [u_t u_t'] = \Sigma_{u_t} = BB'$. In this case, given u_t , the knowledge of the B matrix is sufficient for the correct identification of the shocks.

The variables employed in macroeconomic analysis are often available at different frequencies. For example, financial variables are available at a daily frequency whereas GDP is only at a quarterly one. In this illustration, we consider the simplest possible case where the goal of the empirical analysis is to identify the effect of the innovation ε^y on x : when x is observable only in even periods while y is observable every period. This frequency mismatch (equal to 2) generates an estimation problem as the SVAR in Eq.(1)

is not observable. The problem is commonly solved by temporally aggregating y to the lower frequency at which y is observable. As we show below, this procedure induces a bias in the analysis.

The temporally aggregated system includes the variables at the frequency $\tau = 2, 4, \dots, T$ that correspond to half of the frequency of $t = 1, 2, \dots, T$:⁶

$$\begin{bmatrix} x_\tau \\ y_\tau \end{bmatrix} = \begin{bmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{11}a_{21} + a_{21}a_{22} & a_{12}a_{21} + a_{22}^2 \end{bmatrix} \begin{bmatrix} x_{\tau-1} \\ y_{\tau-1} \end{bmatrix} + \begin{bmatrix} u_\tau^x \\ u_\tau^y \end{bmatrix} \quad (2)$$

where the reduced-form residuals u_τ are a linear combination of the structural ones over periods $t - 1$ and t :

$$\begin{bmatrix} u_\tau^x \\ u_\tau^y \end{bmatrix} = \begin{bmatrix} b_{11}\varepsilon_t^x + (a_{11}b_{11} + a_{12}b_{21})\varepsilon_{t-1}^x + a_{12}b_{22}\varepsilon_{t-1}^y \\ b_{21}\varepsilon_t^x + b_{22}\varepsilon_t^y + (a_{21}b_{11} + a_{22}b_{21})\varepsilon_{t-1}^x + a_{22}b_{22}\varepsilon_{t-1}^y \end{bmatrix} \quad (3)$$

Notice that this process is still a $VAR(1)$ but the variance-covariance matrix of the residuals is affected by this transformation: $\mathbb{E}[u_\tau u_\tau'] = \Sigma_{u_\tau} = (\mathcal{I} + A)BB'(\mathcal{I} + A)'$. It is simple to verify that $\Sigma_{u_\tau} \neq \Sigma_{u_t}$. Thus, even the correct Cholesky structure implicit in B , imposed on u_τ , cannot (in general) recover the shocks ε_τ . In other words, temporal aggregation makes impossible to recover the causal impact of the innovations.

To evaluate the effect of ε_τ^y on x_τ , we have to consider the effect of the innovations occurring in periods $t - 1$ and t . This implies that the impact effects of shock ε^y on the system is the cumulative effect attributable to ε_t^y and ε_{t-1}^y . In a linear model such as the VAR, this is captured by the sum of the innovations $\varepsilon_\tau^y = \varepsilon_t^y + \varepsilon_{t-1}^y$. This is the first crucial

⁶We apply temporal aggregation via skip-sampling over 2 periods, meaning that the variables are observable only once every two periods (and consider the last observable time) but are not transformed. This corresponds to applying the filters $D(L) = I + AL$ and $W(L) = I$ to $[x_t y_t]'$, where L is the lag operator. The Online Appendix reports the same analytical example using averaging as a temporal aggregation filter.

intuition underlying the results in this paper: the linearity of VAR models implies that the sum of HF shocks exerts their cumulative effect because the latter is the sum of their individual effect. However, an additional complication arises from the VAR dynamics. In fact, when evaluating the impact of ε_τ^y on x_τ , ε_t^y exerts only its impact effect on the HF-VAR but ε_{t-1}^y can affect $[x_t \ y_t]'$ during two HF periods. Nonetheless, the second important ingredient of our results is that temporal aggregation preserves the dynamic effects of the HF shocks (within the low-frequency time interval), which are also linear functions of them.⁷ We label $\Theta_{\tau,0}$ the impact effect of the shock ε_τ^y , which is given in this particular case by:

$$\Theta_{\tau,0} = \begin{bmatrix} \Theta_{\tau,0}^x \\ \Theta_{\tau,0}^y \end{bmatrix} = B_{\bullet 2} + (AB)_{\bullet 2} = \begin{bmatrix} a_{12}b_{22} \\ b_{22}(2 + a_{22}) \end{bmatrix} \quad (4)$$

where $B_{\bullet 2}$ and $(AB)_{\bullet 2}$ denote the second column of the matrices B and AB , respectively.

The Bridge-PSVAR maintains the identification at HF and aggregates, as the average or sum, the HF shocks to the lower frequency. Thanks to the two intuitive results previously highlighted, this allows us to correctly identify the impact effect of ε_τ^y on x_τ . We estimate a VAR at the frequency t that includes y_t and a vector of variables Ω_t of length k , which is defined such that the VAR estimated at frequency t allows the identification of the shocks (information sufficiency). The HF-VAR can be estimated using the observables at high-frequency as:

$$\begin{bmatrix} \Omega_t \\ y_t \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \Omega_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} C_{11} & 0 \\ C_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^\Omega \\ \varepsilon_t^y \end{bmatrix} \quad (5)$$

⁷The temporal aggregation filter correctly preserves the dynamic effects of the HF shocks between horizon 1 and m and consequently also the aggregated impact effect at LF. Conversely, the LF-VAR autoregressive components are employed to compute the dynamic effects at LF.

where $D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & d_{22} \end{bmatrix}$ is the autoregressive matrix and $C = \begin{bmatrix} C_{11} & 0 \\ C_{21} & c_{22} \end{bmatrix}$ is the contemporaneous one. The informational sufficiency assumption states that $\varepsilon_t^\Omega \supseteq \varepsilon_t^x$ and, under this condition and with the right identification strategy, we recover ε_t^y or at least its proxy (i.e. a noisy measure of the shock) $z_t = \varepsilon_t^y + \eta_t, \eta_t \sim wn(0, \sigma_\eta^2), \eta_t \perp \varepsilon_t^y$.

In this illustrative case, $\{z_{t-1}, z_t\}$ are aggregated to z_τ as the average over τ :

$$z_\tau = \frac{z_{t-1} + z_t}{2} \quad \{\tau, t\} = 2, 4, \dots, T \quad (6)$$

Then, we employ z_τ as a proxy for the structural shock ε_τ^y . This strategy is valid under the typical assumption in the Proxy-SVAR literature on the exogeneity of z_t , i.e. $\mathbb{E}[z_t \varepsilon_t^x] = 0$, and its strength, i.e. $\mathbb{E}[z_t \varepsilon_t^y] \neq 0$. These two properties are translated to ε_τ^y under the correct specification of the HF-VAR ensured by our assumptions on Ω_t (and a large enough number of lags compared to the the frequency mismatch). In this way, we correctly identify the impact effect of the shock ε_τ^y up to a scale factor μ :

$$\Theta_{\tau,0}^y = \mathbb{E}[z_\tau z_\tau]^{-1} \mathbb{E}[z_\tau u_\tau^y] = \mu (2b_{22} + a_{22}b_{22}) \quad (7)$$

$$\Theta_{\tau,0}^x = \mathbb{E}[z_\tau z_\tau]^{-1} \mathbb{E}[z_\tau u_\tau^x] = \mu a_{12}b_{22} \quad (8)$$

In this way, the ratio $\frac{\Theta_{\tau,0}^x}{\Theta_{\tau,0}^y}$ is correctly estimated (see Eq. 4).

Notice that the aggregation of the shocks in alternative ways would not yield the correct ratio $\frac{\Theta_{\tau,0}^x}{\Theta_{\tau,0}^y}$. For example, in the literature, shocks available at a daily frequency have been sometimes aggregated by using weights proportional to the days left in a month or as moving averages. Our paper shows that those approaches are inconsistent with an underlying VAR structure.

3 Methodology

This section formalizes and generalizes the description of the Bridge Proxy-SVAR. Section 3.1 presents the econometric framework used for the analysis, then Sections 3.2 to 3.4 describe the different steps of the methodology in detail.

The framework that we consider is the following: i) the vector of innovations that drives the variables can be partitioned as $\varepsilon = \begin{bmatrix} \varepsilon^1 & \varepsilon^{\bar{1}} \end{bmatrix}$, where ε^1 denotes the structural shock of interest and $\varepsilon^{\bar{1}}$ is the vector that comprises all the other shocks of the system; ii) the objective is to identify the effect the innovation ε^1 on a vector of endogenous variables y ; iii) ε^1 can be recovered at the frequency $t = 1, 2, \dots, T$ higher than the frequency τ at which y is observable ($\tau = m, 2m, \dots, T$ where $m > 1$ is the frequency mismatch). Therefore the Bridge-PSVAR is comprised of three steps: 1) identify ε_t^1 in a daily VAR; 2) correctly aggregate ε_t^1 transforming it into ε_τ^1 ; and 3) estimate a VAR at lower frequency on y_τ and use ε_τ^1 as external instrument to estimate the causal impact effect of ε^1 on y .

3.1 Econometric Framework

Consider a vector of n time series modeled as a causal and covariance stationary SVAR of lag-length p :

$$y_t = A_1 y_{t-1} + \dots + A_p y_{t-p} + B \varepsilon_t \quad (9)$$

where ε_t is a vector of stochastic innovations and B is a $n \times n$ matrix whose coefficients determine how ε_t contemporaneously affects the variables y_t . Such a process can be expressed via compact notation through the polynomial $A(L) = \mathcal{I} - A_1 L - A_2 L^2 - \dots - A_p L^p$:

$$A(L)y_t = B\varepsilon_t \quad (10)$$

where L is the lag operator such that $L^i y_t = y_{t-i}$. The SVAR is not observable itself, but corresponds to a multiplicity of reduced-form VAR representations of the form:

$$A(L)y_t = u_t \quad (11)$$

In what follows we focus exclusively on the problem of identification of matrix B under temporal aggregation. Temporal aggregation can be expressed as a two-step filter. First, the data are made observable only once every m periods, which represents the frequency mismatch, via the filter $D(L) = \mathcal{I} + D_1 L + D_2 L^2 + \dots + D_{pm-p} L^{pm-p}$. The specification of $D(L)$ has to be such that the elements of $D(L)A(L)$ are powers of L^m , meaning that only the observable data points enter the transformed process. The conditions for the existence of such a filter, as well as the values taken by the matrices D_i are derived in [Marcellino \(1999\)](#). The second filter, denoted by $W(L)$, depends on the temporal aggregation scheme considered; skip-sampling (or point-in-time sampling) is usually applied to stock variables (e.g. prices) whereas averaging is typically applied to flow variables (e.g. volumes). In the former case, $W(L)$ does not modify the original data, i.e. $W(L) = \mathcal{I}$. For example, consider the $VAR(1)$ process analyzed in the stylized example: $y_t = A_1 y_{t-1} + B\varepsilon_t$ and $m = 2$. The filter $D(L)W(L) = \mathcal{I} + A_1 L$ transforms the original process into $(\mathcal{I} + A_1 L)y_t = (\mathcal{I} + A_1 L)A_1 y_{t-1} + (\mathcal{I} + A_1 L)B\varepsilon_t$, which can be rearranged as $y_\tau = A_1^2 y_{\tau-1} + B\varepsilon_t + A_1 B\varepsilon_{t-1}$. In the averaging case $W(L) = \mathcal{I} + L + L^2 + \dots + L^{m-1}$ and for the specific process under consideration $W(L) = \mathcal{I} + L$ and $D(L)W(L) = (\mathcal{I} + A_1 L)(\mathcal{I} + L)$. The temporally aggregated process would then become $\bar{y}_\tau = A_1^2 \bar{y}_{\tau-1} + B(\varepsilon_t + \varepsilon_{t-1}) + A_1 B(\varepsilon_{t-1} + \varepsilon_{t-2})$ with $\bar{y}_\tau = y_\tau + y_{\tau-1}$.

The typical object of interest in the SVAR literature are the dynamic effects of the innovations ε_t on y_{t+k} where $k \in \mathbb{N}$ represents the horizon. The impact effects of the innovations are defined as: $\Theta_{0,t} = \mathbb{E}_{\mathcal{F}_{t-1}}[y_t/\varepsilon_t = 1] - \mathbb{E}_{\mathcal{F}_{t-1}}[y_t/\varepsilon_t = 0]$ with the information set $\mathcal{F}_{t-1} = \{y_{t-1}, \dots, y_{t-p}\}$. Conversely, there are two possible definitions of temporally aggregated IRFs $\Theta_{0,\tau}$ according to the information set considered at LF. A first possibility employs the relevant information set for the LF representation; the information set is $\mathcal{F}_{\tau-1} = \{y_{\tau-1}, y_{\tau-2}, \dots\}$ and consequently $\Theta_{0,\tau} = \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_\tau/\varepsilon_\tau = 1] - \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_\tau/\varepsilon_\tau = 0]$ where $\varepsilon_\tau = \{\varepsilon_{t-m+1}, \varepsilon_{t-m+2}, \dots, \varepsilon_t\}$ for $\{t, \tau\} = m, 2m, \dots, T$. This definition considers all the innovations occurring between $\tau - 1$ and τ . Alternatively, the LF-IRFs can be defined directly via the temporal aggregation filter, implicitly using the information set $\mathcal{F}_{t-1}$ (i.e. using the HF information set). In this case, the LF-IRFs are defined directly as $\Theta_{0,\tau} = D(L)W(L)\Theta_0^t$. Thus, this choice is key to determining how to correctly use the HF shocks ε_t . While both definitions are formally correct, we regard the first as the most interesting from a macroeconomic perspective. Thus we focus on this definition of LF-IRFs and provide the discussion of the LF-IRFs defined by the temporal aggregation filters in the Appendix. [Chudik and Georgiadis \(2019\)](#) focus instead on the LF-IRFs as defined by the temporal aggregation filter. They propose a mixed frequency regression (MIDAS) to estimate the effect of shocks already available at a higher frequency (for example, based on HF event-studies) than endogenous variables. The Bridge-PSVAR also differs from their approach since it identifies the shock of interest using a HF-VAR and applies the correct aggregation filter to transform shocks from HF to LF.

3.2 First Step: Identification at High-Frequency

The first step concerns the identification of a shock or proxy ε_t^1 at the high-frequency $t = 1, 2, \dots, T$. The HF-VAR has to be specified to achieve informational sufficiency, meaning

that the shock ε_t^1 can be recovered as a linear combination of the reduced form residuals if the appropriate identification strategy is applied. Thus, a set of variables Ω_t are included to achieve this goal as in Eq.(12). $\mathring{A}(L)$ and $\mathring{B}$ denote respectively the autoregressive matrix and the impact matrix that characterize the HF system $\begin{bmatrix} \Omega & y_t^1 \end{bmatrix}'$:

$$\mathring{A}(L) \begin{bmatrix} \Omega_t \\ y_t^1 \end{bmatrix} = \mathring{B} \begin{bmatrix} \varepsilon_t^\Omega \\ \varepsilon_t^1 \end{bmatrix} \quad (12)$$

After estimating the reduced form HF-VAR, one can apply any of the different identification strategies previously used by the literature to identify the structural shock of interest from the reduced form residuals (i.e. recursive, sign, or narrative restrictions).⁸ The identification at HF recovers z_t^1 , a potentially noisy measure of the shock ε_t^1 as described in Eq.(13):

$$z_t^1 = \varepsilon_t^1 + w_t \quad (13)$$

where w_t is a measurement error such that $\mathbb{E}[w_t w_t] = \sigma_w^2$ and $\mathbb{E}[\varepsilon_t^1 w_t] = 0$. Assuming that $\sigma_w^2 = 0$ implies that we are recovering the true shock. In what follows, we use z_t and ε_t interchangeably.

3.3 Second Step: Aggregation and Information Sufficiency Test

In the second step, we aggregate the shocks identified at high-frequency and test for their partial invertibility. Let $\varepsilon_\tau = \{\varepsilon_{t-m+1}, \varepsilon_{t-m+2}, \dots, \varepsilon_t\}$ for $\{t, \tau\} = m, 2m, \dots, T$ such that $\varepsilon_\tau = 1$ implies $\{\varepsilon_{t-m+1} = 1, \varepsilon_{t-m+2} = 1, \dots, \varepsilon_t = 1\}$. Considering that the IRFs are defined in terms of the LF information set $\mathcal{F}_\tau$ (see Section 3.1 for a discussion on this issue), the impact response of the system is given by $\Theta_{0,\tau} = \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_\tau / \varepsilon_\tau = 1] - \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_t / \varepsilon_\tau = 0]$.

⁸See Ramey (2016) or Kilian and Lutkepohl (2017) for a summary of different identification strategies

This means that the evaluation of the IRFs takes into account all the innovations occurring between $\tau-1$ and τ . Then we define $\{\Gamma_{m-1}, \Gamma_{m-2}, \dots, \Gamma_0\}$ as the sequence of the LF impact effects associated respectively to the shocks $\{\varepsilon_{t-m+1}, \varepsilon_{t-m+2}, \dots, \varepsilon_t\}$. For example, in the stylized process of Section 2 $\Gamma_0 = \Theta_{0,t} = B$ and $\Gamma_1 = \Theta_{0,t} + A_1 \Theta_{0,t} = (\mathcal{I} + A_1) B$. Then the aggregated response at LF is given by $\Theta_{0,\tau} = \sum_{i=0}^{m-1} \Gamma_i$ and can be recovered according to Proposition I.

Proposition I. *Let y_t be an underlying HF-VAR process, which is temporally aggregated as a LF-VAR y_τ through the filters $D(L)$ and $W(L)$. u_τ denotes the reduced form residuals obtained from the estimation of the LF-VAR. Then, the IRF $\Theta_{0,\tau}$ is recovered by projecting u_τ on the average of the HF shock that occurred within the LF period $\varepsilon_\tau = \frac{\sum_{i=1}^m \varepsilon_{t-i}}{m}$ for $\{t, \tau\} = m, 2m, \dots, T$. Thus the correct filter $J(L)$ applied to ε_t is $J(L) = I + L + \dots + L^{m-1}$.*

Proof: see Appendix B. ■

Proposition I relies on the linearity of VAR models: linearity implies that the sum of the causal effect of the HF shocks $\varepsilon_\tau = \{\varepsilon_{t-m+1}, \varepsilon_{t-m+2}, \dots, \varepsilon_t\}$ for $\{t, \tau\} = m, 2m, \dots, T$ is equal to the causal effect of their sum, i.e. the aggregated LF shock ε_τ . According to Proposition I, we define the LF aggregate of z_t as $z_\tau = \sum_{i=1}^m z_{t-i}$ for $\{t, \tau\} = m, 2m, \dots, T$.⁹

The crucial property that distinguishes structural shocks from reduced form residuals, i.e. their orthogonality, is preserved when the HF shocks are aggregated to LF according to Proposition II. It is worth stressing that the proposition provides sufficient conditions for the orthogonality of the LF shocks but not a necessary condition.

Proposition II. *Given the shocks $\varepsilon_t = [\varepsilon_t^1, \varepsilon_t^{\bar{1}}]$ identified in the HF-VAR that by construction satisfy $\mathbb{E} [\varepsilon_t^1 \varepsilon_t^{\bar{1}'}] = 0$, if the HF-VAR approximates well enough the data generating process, then $\mathbb{E} [\varepsilon_\tau^1 \varepsilon_\tau^{\bar{1}'}] = 0$.*

⁹We do not deal explicitly with HF shocks coming from narrative sources and event-studies. However, our theoretical results showing the correct aggregation of shocks could be relevant for this strand of literature. For example, past works have aggregated HF shocks to lower frequencies by taking their moving average or a weighted average with weights proportional to the remaining days within the month. However those procedures are inconsistent if the underlying model is a HF-VAR.

Proof: see Appendix B. ■

An ancillary result regarding the autocorrelation of the aggregated shocks, which is based on Proposition II, is contained in Lemma I.

***Lemma I.** Given the shocks ε_t identified in the HF-VAR, if the HF-VAR is a good enough approximation of the data generating process, then the shocks aggregated at the LF ε_τ do not display autocorrelation.*

Proof: see Appendix B.

Information Sufficiency Test

Our HF identification is performed in a system that does not include the low-frequency endogenous variables. For example, in case of a daily-monthly frequency mismatch, this means that macroeconomic variables are not employed for identification but just in the third step of the Bridge-PSVAR as endogenous variables. The presence of endogenous macroeconomic variables is neither a necessary nor a sufficient condition to achieve a correct identification; any small scale VAR may suffer the same problem of potential omitted variables. In fact, the literature has proposed a number of tests to detect whether omitted variables contaminate the identified shocks. The most popular is the [Forni and Gambetti \(2014\)](#) sufficient information test. Such test is discussed in the framework of external instruments both in [Stock and Watson \(2018\)](#) and [Miranda Agrippino and Ricco \(2018\)](#).

In practice, following [Forni and Gambetti \(2014\)](#), we regress z_τ^1 on the lags of factors or principal components Λ of large datasets. The following relationship should hold:

$$\mathbb{E} \left[\Lambda_{\tau-k} z_\tau^1 \right] = 0 \quad \forall k \in \mathbb{N}^+ \quad (14)$$

This test assures that the shocks are not predictable and thus are proper structural

shocks. Specific variables, for instance those endogenous variables of LF-VAR, can also be employed in this test instead of factors to assess whether the identified shock is contaminated by past macroeconomic variables.

3.4 Third Step: External Instruments and Identification of the LF-VAR

The last step is twofold. First, we estimate the LF-VAR of order p in Eq.(15) including the relevant HF variables (aggregated at LF) together with the macroeconomic variables of interest in the system:

$$y_\tau = \tilde{A}_1 y_{\tau-1} + \tilde{A}_2 y_{\tau-2} + \dots + \tilde{A}_p y_{\tau-p} + u_\tau \quad (15)$$

where u_τ denotes the vector of reduced form residuals. Second, the causal impact effect of ε_τ^1 on y_τ is identified by employing z_τ^1 as external instrument. Assuming that the shock of interest ε_τ^1 is invertible, i.e. it can be expressed as a linear combination of the reduced form residuals u_τ , inference from the Proxy-SVARs is valid under three conditions:

- i) exogeneity: $\mathbb{E} \left[\varepsilon_\tau^{\bar{1}} z_\tau^1 \right] = 0$
- ii) strength: $\mathbb{E} \left[\varepsilon_\tau^1 z_\tau^1 \right] \neq 0$
- iii) limited lag-lead exogeneity: $\mathbb{E} \left[e_{\tau+j}^{\bar{1}} z_\tau^1 \right] \neq 0$ for $j \neq 0$, where $e^{\bar{1}}$ denotes the subset of $\varepsilon_\tau^{\bar{1}}$ of non-invertible shocks.

The first condition implies that the proxy has to be (contemporaneously) uncorrelated with the other structural shocks of the system. This condition, analogous to the exclusion restriction for the IV estimator, cannot be tested directly and rests on the validity of the identification assumptions. The second condition is related to the relevance of the instrument. [Montiel Olea et al. \(2018\)](#) argue that the robust first stage

F-stat can be compared to the [Stock and Yogo \(2005\)](#) critical values. The third condition states that the proxy should be uncorrelated to lags and leads of the non-invertible shocks affecting the system. These three conditions are discussed in detail in [Stock and Watson \(2018\)](#) and [Miranda Agrippino and Ricco \(2018\)](#).

By projecting u_τ on z_τ^1 the relative impulse responses are correctly identified:

$$\Theta_0^\tau \propto \mathbb{E} \left[u_\tau z_\tau^1 \right] \quad (16)$$

Under these conditions the external instrument approach is statistically efficient because it does not estimate additional parameters. z_τ^1 could be also employed as an internal instrument or as an exogenous variable in the LF-VAR.

The correct way to use external information to estimate the relevant statistics like IRFs and forecast error variance has been analyzed in the literature (see, for example, [Plagborg-Møller and Wolf, 2019](#); [Noh, 2018](#); [Miranda Agrippino and Ricco, 2018](#); and [Paul, forth](#)). The shock captured by the proxy is partial invertible if and only if the proxy does not Granger cause the residuals of the LF-VAR. If the test is not passed, then the inference based on the Proxy-SVAR is not valid but the relative IRFs can be still estimated by including the proxy as an exogenous variable in the VAR ([Paul, forth](#)). Alternatively, the proxy can be used as an internal instrument in the VAR by including z_τ^1 as an endogenous variable in the VAR and computing the IRFs by ordering the proxy first in a Cholesky decomposition (see, for example, [Plagborg-Møller and Wolf, 2019](#); [Noh, 2018](#); and [Miranda Agrippino and Ricco, 2018](#)). These latter approaches constitute a parsimonious equivalent of the popular local projection instrumental variable approach (LP-IV) with controls. Notice that, under the testable assumption (14), the lagged endogenous variables of the LF-VAR are orthogonal to the proxy, thus the impact effect estimated using the proxy as external instrument, as internal instrument, and as

exogenous variable in the VAR coincide. Therefore, the partial invertibility of the shock of interest and the consistent modeling strategy to compute the IRFs matters only when estimating the dynamic effects of the shocks of interest.

The confidence bands for the IRFs can be computed using different methods. While wild bootstrap is the most popular (see, for example, [Gertler and Karadi, 2015](#); [Mertens and Ravn, 2018](#)), [Jentsch and Lunsford \(2016\)](#) show that this method is asymptotically invalid and propose a residual-based moving block bootstrap procedure. [Montiel Olea et al. \(2018\)](#) employ instead the Delta method to compute the confidence sets. The differences between these methods become larger when the F-statistic for the instrument is lower (see, for example, [Mertens and Ravn, 2019](#)).

4 Monte Carlo Experiments

4.1 Experimental Design

We rely on Monte Carlo experiments to test the performance of the Bridge-PSVAR in identifying the correct impact matrix in a general setup and finite samples. Within the previous econometric framework, we compare the performances of the HF-VAR (high-frequency data), LF-VAR (time aggregated data), and the Bridge-PSVAR in recovering the underlying DGP. The HF-VAR is a counter-factual exercise where all the variables are observable at HF and by construction it recovers the correct impact effect. The Bridge-PSVAR identifies the shocks in a HF system that is informationally sufficient and then uses the aggregated shock to instrument the reduced form residual of the LF-VAR. Finally, the LF-VAR identifies the shock by applying the same identification scheme but in a temporally aggregated system. The LF-VAR and the Bridge-PSVAR temporally aggregate information in opposite ways: in a LF-VAR the aggregation occurs

before the identification, whereas the Bridge-PSVAR identifies the shocks at HF and then correctly filters them at LF. By applying the correct filter, our methodology correctly recovers the impact's causal effects. Conversely, the LF-VAR identifies IRFs that generally diverge from those implied by the data generating process.

Our experimental design is similar to that in [Foroni and Marcellino \(2016\)](#). The DGP described in Eq.(17) is a $VAR(1)$ process driven by Cholesky innovations such that the different methodologies use the same (correct) identification scheme.

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{22} & a_{21} \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} e_t^x \\ e_t^y \end{pmatrix} \quad (17)$$

where $\begin{pmatrix} e_t^x \\ e_t^y \end{pmatrix} \sim \mathcal{N}(0, \mathcal{I}_2)$. The simulations consider the cases of temporal aggregation via skip-sampling and averaging for both the frequency mismatch m equal to 3 (monthly-quarterly case) and to 30 (daily-monthly case). We compare two different LF sample sizes $\mathcal{T}$ to take both short and large samples into account. Specifically we set $\mathcal{T} = 100$ and $\mathcal{T} = 1000$. The autoregressive coefficients are drawn from a uniform distribution as $\{\rho_l, \rho_h, \delta_l, \delta_h\} \sim \mathcal{U}(-1, 1)$. We select those parametrizations with real eigenvalues that belong to the set $(0.7, 0.95)$ to avoid non-stationarity and to impose some persistence in the IRFs.¹⁰ The impact coefficients are drawn as $\{b_{11}, b_{22}\} \sim \mathcal{U}(0, I_2)$ and $b_{12} \sim \mathcal{U}(-1, 1)$. We retain those parametrizations that satisfy $b_{21} < b_{11}$ and $b_{21} < b_{22}$ to maintain a mapping between shocks and variables. For the same reason, we impose $b_{11} > 0.1$ and $b_{22} > 0.1$. In order to provide a representative measure of the parameter space, we repeat the simulations for 100 random parametrizations.

Each experiment is repeated over 1000 simulations and the performances of the

¹⁰This is important because IRFs aggregated at LF are an uninteresting case without persistence, yielding zero effect at LF independently of the impact matrix.

different methods are evaluated via the cumulative *Mean Absolute Distance* (MAD) at 8 horizons between the true IRFs and the estimated one. Notice that our synthetic measure takes into account the precision of the estimates as the MAD is computed for each replication and then is averaged over the whole set of replications. The MAD is defined as:

$$mad_{i,j}^{id} = \sum_{h=1}^8 \left| \Theta(h)_{i,j} - \hat{\Theta}^{id}(h)_{i,j} \right| \quad i, j = \{1, 2\}$$

for $id = \{\text{HF-VAR}, \text{LF-VAR}, \text{Bridge-PSVAR}\}$, where $\Theta(h)_{i,j}$ denotes the response of variable j to shock i at the horizon h . This metric is then aggregated for each parametrization k , over variables j and shocks i as $MAD_k^{id} = \sum_{i=1}^2 \sum_{j=1}^2 mad_{i,j}^{id}$. Finally, a unique metric across all parametrizations is obtained as $MAD^{id} = \sum_{k=1}^{100} MAD_k^{id}$.

4.2 A Specific Parametrization

Before presenting the general performances of the methodologies, this section graphically illustrates the results from a specific parametrization of the DGP. We select

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{22} & a_{21} \end{pmatrix} = \begin{pmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{pmatrix}, \quad \begin{pmatrix} b_{11} & 0 \\ b_{12} & b_{21} \end{pmatrix} = \begin{pmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{pmatrix}$$

for the parameters of Eq.(17). For ease of exposition, we focus on the temporal aggregation via skip-sampling for a monthly-quarterly case ($m = 3$) with $\mathcal{T} = 1000$. Figures 1 and 2 display the IRFs of the system to a shock e_t^x and to a shock e_t^y , respectively.

There are significant differences in the estimated IRFs by the HF-VAR (blue continuous line), the Bridge-PSVAR (red diamond line), and the LF-VAR (green plus line), together with the IRFs implied by the DGP (black). The HF-VAR perfectly recovers

the IRFs, whereas the LF-VAR completely misses the shape and sign of the impact effects of e^x on y , which in turns also implies a wrong estimated dynamic effect on x itself. The LF-VAR constraints the impact of e^y on x to be 0, thus missing the actual negative effect. The Bridge-PSVAR closely replicates the performances of the HF-VAR but it is less efficient being a two stage estimation (IV versus OLS).¹¹

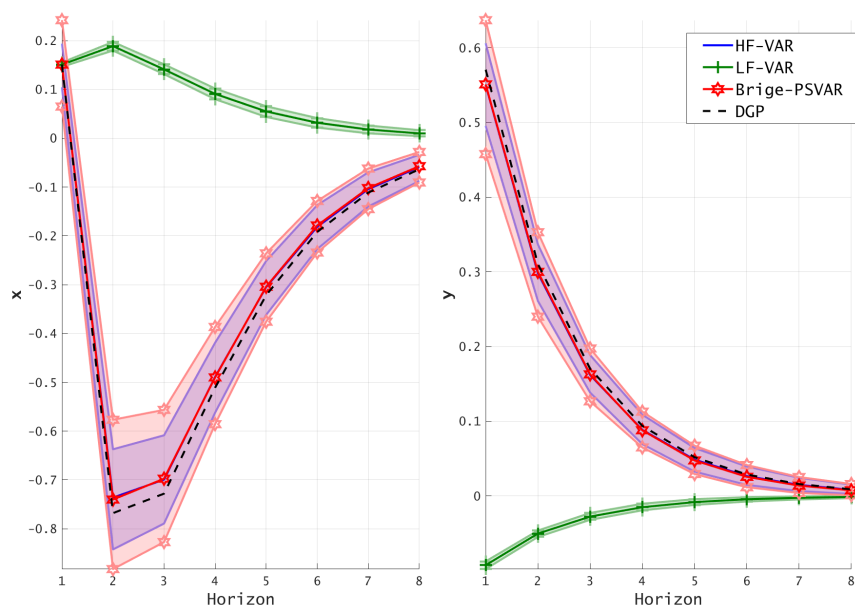


Figure 1: IRF to ε^x - MONTE CARLO EXPERIMENT

IRFs to a shock in the first variable of the bivariate system (x). The true IRF is represented by the dotted black line. The autoregressive matrix is $\begin{bmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{bmatrix}$ and the impact matrix is $\begin{bmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{bmatrix}$. The shock is identified through the correct recursive structure in the HF system (blue), the LF system (green) and the Bridge-PSVAR (red). Shaded areas correspond to the 90% percentile across 1000 replications. Time aggregation follows a skip-sampling scheme and the frequency mismatch is 3 (monthly-quarterly case).

¹¹The impact IRFs diverge from the B matrix due to the definition of LF-IRFs that aggregates the effects of all of the HF shocks.

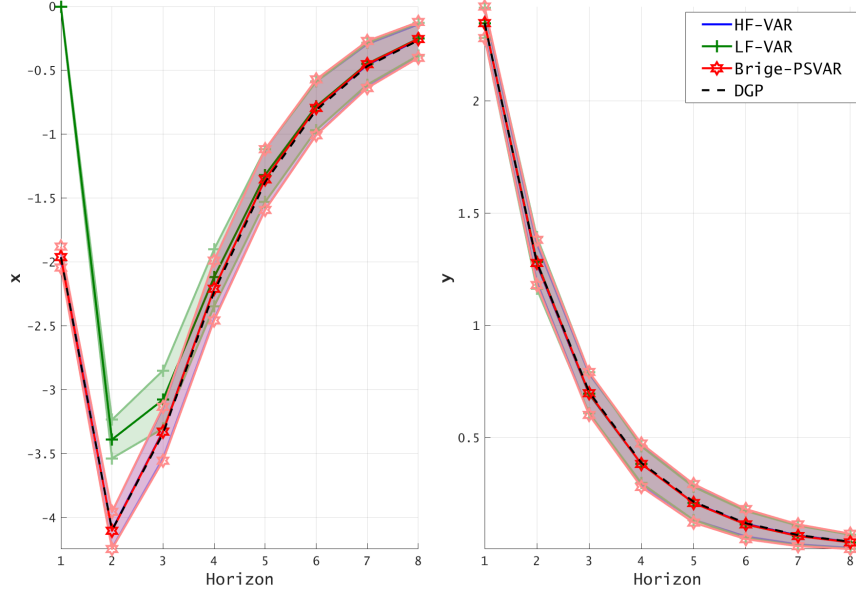


Figure 2: IRF to ε^y - MONTE CARLO EXPERIMENT

IRFs to a shock in the second variable in the bivariate system (y). The true IRF is represented by the dotted black line. The autoregressive matrix is $\begin{bmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{bmatrix}$ and the impact matrix is $\begin{bmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{bmatrix}$. The shock is identified through the correct recursive structure in the HF system (blue), the LF system (green) and the Bridge-PSVAR (red). Shaded areas correspond to the 90% percentile across 1000 replications. Time aggregation follows a skip-sampling scheme and the frequency mismatch is 3 (monthly-quarterly case).

4.3 General Results

This section describes the general results from our Monte Carlo simulations. Our goal is to provide a synthetic measure of the accuracy differences across the three methodologies in estimating IRFs. For this reason, we sum the (replication mean) MAD errors for each IRF (four in total, two for each of the two shocks) for each of the 100 parametrizations of the DGP. This absolute measure is transformed into a relative summary statistics as the MAD percentage reduction compared to the LF-VAR that can be expressed as $\tilde{MAD}^{id} = \frac{MAD^{id} - MAD^{LF}}{MAD^{LF}}$ for $id = \{HF-VAR, Bridge-PSVAR\}$. Table 1 reports a synthetic measure across the following crucial features of the experiment: the temporal aggregation scheme (skip-sampling or averaging), the sample size at LF (small - 100 or large - 1000), and the frequency mismatch (3 or 30 representing the monthly-quarterly or daily-monthly cases,

respectively).¹²

The HF-VAR represents the first-best procedure and the corresponding MAD gains constitute thus an upper bound in the accuracy of the estimation. Our simulations confirm that the informational sufficient Bridge-PSVAR, which applies the appropriate temporal aggregation filter to the HF shocks, correctly recovers the contemporaneous matrix of the SVAR. The lower MAD gains reported for the Bridge-PSVAR are due to the loss of accuracy in a two-stage approach, which resembles a standard loss of efficiency when using IV estimation compared to OLS. Nonetheless, the MAD gains are sizable and in many cases quite close to the performances of the HF-VAR, which constitutes the (counterfactual) upper bound. All in all, the results from these simulations confirm and complement the theoretical results from Section 3, validating our methodology. In the next section, we illustrate its advantages compared to identification in LF-VARs through an empirical application.

¹²Notice that whereas a skip-sampled VAR(1) remains a VAR(1), temporal aggregation by averaging transforms the process into a VARMA. Consequently, the estimated autoregressive matrix of the LF-VAR is biased but this bias decreases by the sample size. On the one hand, the impact effect identified by the Bridge-PSVAR is not affected by the misspecification of the LF-VAR. On the other hand, the biased autoregressive matrix is common across the LF-VAR and the Bridge-PSVAR when computing the dynamic effects.

MAD gains over LF-VAR		
	Skip-sampling	Averaging
Small (LF) sample size T=100		
<i>Frequency Mismatch: Monthly-Quarterly Case (3)</i>		
HF-VAR	65%	83%
Bridge-PSVAR	59%	19%
<i>Frequency Mismatch: Daily-Monthly Case (30)</i>		
HF-VAR	80%	97%
Bridge-PSVAR	43%	42%
Large (LF) sample size T=1000		
<i>Frequency Mismatch: Monthly-Quarterly Case (3)</i>		
HF-VAR	87%	95%
Bridge-PSVAR	85%	21%
<i>Frequency Mismatch: Daily-Monthly Case (30)</i>		
HF-VAR	93%	90%
Bridge-PSVAR	79%	79%

Table 1

Performance comparisons across the counter-factual HF-VAR, the LF-VAR, and the Bridge-PSVAR. Performances are evaluated in terms of the Mean Absolute Distance (MAD) between the true IRFs and the estimated IRFs in 100 randomly parametrized DGPs. One summary statistic is computed as a mean across all combinations of shocks-variables in the system. The gains are expressed as percentage MAD gains over the LF-VAR. We analyze different cases for a VAR(1) DGP by varying: i) temporal aggregation scheme, either skip-sampling or averaging; ii) the frequency mismatch between HF and LF by 3 (monthly-quarterly case) or 30 (monthly-daily case); iii) sample size, either small (100 LF observations) or large (1000 LF observations).

5 Empirical Applications

5.1 Uncertainty Shocks

The macroeconomic effects of uncertainty shocks have recently been the subject of considerable debate both on the theoretical side and on the empirical side. Recent theoretical papers show that the effects of uncertainty shocks are rather ambiguous even when simply considering the sign of the output response.¹³ Berger et al. (2019) (BDG henceforth) argue that the major identification problem faced by the literature is that uncertainty about the future is positively correlated with current economic developments, which are reflected in the equity market as realized volatility. Based on

¹³Basu and Bundick (2017) and Bloom et al. (2018), among others, discuss the compatibility of their models with both recessionary and expansionary effects of uncertainty shocks.

this fact, they aim to isolate exogenous fluctuations in uncertainty by assuming that uncertainty shocks do not affect realized volatility within the same month. BDG employ a VAR model of the U.S. economy, estimated over the sample 1986:1m-2014:12m, that includes realized volatility (rv), option-implied volatility (v_1 - basically the VIX), the fed fund rate (ffr), industrial production (ip), and employment (emp); where v_1 contains crucial information to predict rv . They identify uncertainty shocks as the linear combination of the reduced form residuals that maximizes the two-year ahead forecast error variance (FEV) of realized volatility but that does not affect realized volatility within the same month (MFEV henceforth), following the identification strategy used to identify TFP news shocks (see, for example, Barsky and Sims, 2011). Although uncertainty shocks account for 30-60 percent of the FEV of realized volatility, they do not induce significant macroeconomic effects after controlling for contemporaneous changes in realized volatility. This finding contradicts previous works that found uncertainty shocks to be significantly recessionary (see, for example, Piffer and Podstawski, 2018). BDG instead find that realized volatility shocks (i.e. surprises in realized volatility) are the ones that produce recessionary effects.

We revise the results of BDG on the effects of uncertainty shocks by applying the Bridge-PSVAR, that combines HF identification with LF endogenous variables. In particular, the subdued role of uncertainty in explaining business cycle fluctuations may be driven by the assumption that realized volatility cannot move within the same month in response to uncertainty shocks. In order to test this hypothesis, we use the same dataset as BDG but apply their identification strategy in daily VAR instead. Then, we estimate the macroeconomic effects of uncertainty and realized volatility shocks by means of a monthly VAR. Thus, we are employing a less restrictive identification restriction since we impose that uncertainty shocks do not affect realized volatility only within the same day (compared to one month for BDG).

As a preliminary step, we repeat at daily frequency the predictive regression run by BDG on monthly frequency, where realized volatility is proxied by the daily squared returns in the equity market. Specifically, we regress future realized volatility rv cumulated over the leads in the next 6 months (504 days in our case), i.e. $\sum_{i=1}^{504} rv_{t+i}$, on current $v_{1,t}$. The regression yields $R^2 = 0.46$, a nearly identical result to those performed in BDG at the monthly frequency (Table 2 - column 1). We then estimate a daily VAR on rv and v_1 and apply the identification scheme proposed by BDG (including the same horizon for the MFEV). Then, we aggregate the two series of shocks as monthly averages and test their orthogonality to lagged factors extracted from the FRED-MD database. The test cannot reject that the two shocks are not contaminated by previous innovations (Table 3 - column 1). Thus, we discard the bivariate daily specification and include the following variables in our daily VAR: *s&p500*, Fed Funds rate, commodity price index, BBA corporate spread, euro-dollar exchange rate, economic policy uncertainty index, gold price, 1y Treasury yield, mortgage rate, and term premia (1y, 2y, 6y, and 10y). We repeat the same steps as before and update the forecasting regression with the new daily VAR. Column (2) in Table 2 documents that the R^2 increases from 0.46 to 0.68. Column (2) in Table 3 reports the results from the information sufficiency test, which is passed by the shocks identified using our richer specification. Therefore, we proceed using the aggregated series of shocks as external instruments for the monthly VAR as specified in BDG.

Figure 3 compares the response of the system to our uncertainty shocks identified with the Bridge-PSVAR (left column - red color) vis-à-vis the BDG results (right column - green color) based on the same identification strategy applied at a monthly frequency.¹⁴ First, the zero restriction imposed in BDG at the monthly frequency does not hold

¹⁴We consider their unrestricted VAR specification for comparability reasons. Results are very close to their benchmark restricted specification.

according to our milder identification scheme since realized volatility significantly increases in response to an uncertainty shock. Second, the (stronger) monthly constraint is essential for their conclusions on the null macroeconomic effects of uncertainty shocks. In fact, contrary to BDG, uncertainty shocks have statistically significant effects on ip and emp , which characterizes them as recessionary. Figure 4 displays the monthly IRFs to the uncertainty and realized volatility shocks. We apply the same normalization as BDG such that the two shocks have the same cumulated impact on rv over horizons 2-24. While we reject the conclusion on the null effects of uncertainty shocks, we confirm the BDG conclusion that realized volatility shocks produce on average larger economic effects on both industrial production and employment.¹⁵

	(1)	(2)
	$\sum_{i=1}^{504} rv_{t+i}$	$\sum_{i=1}^{504} rv_{t+i}$
$v_{1,t}$	0.68*** (0.01)	0.47*** (0.01)
Additional predictors	×	✓
Observations	7181	7181
$R^2 - adj$	0.46	0.68

Table 2: Predictive Regressions

Predictive regressions of 6-months rv . (1) includes only v_1 as regressor; (2) includes all variables included in the full VAR specification.

*Standard errors are reported in parenthesis, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.*

¹⁵In a self-contained but related work, [Alessandri et al. \(2019\)](#) propose an alternative identification strategy of uncertainty shocks with the Bridge-PSVAR. The work identifies uncertainty shocks from equity market data as those fluctuations in the vxo that are independent of first-moment shocks (sp) and without assuming that they have no impact effect on sp within the same day.

	(1) Bivariate DVAR		(2) Full DVAR	
	ε_{τ}^u	ε_{τ}^{rv}	ε_{τ}^u	ε_{τ}^{rv}
$F - stat_{(8,338)}$	2.42**	1.10	1.10	0.41
$F - test(pval)$	0.01	0.36	0.38	0.89
Observations	347	347	347	347
$R^2 - adj$	0.06	0.07	0.01	0.01

Table 3: Invertibility Test - Uncertainty Shocks

The regressions include seven lagged factors from FRED-MD database, one lag of the dependent, and a constant. ε_{τ}^u and ε_{τ}^{rv} denote the uncertainty and realized volatility shocks. F -stat denotes the value of the F test statistic and F -test is the p value on the joint test of all coefficients associated with the factors being 0. Standard errors are reported in parenthesis, *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

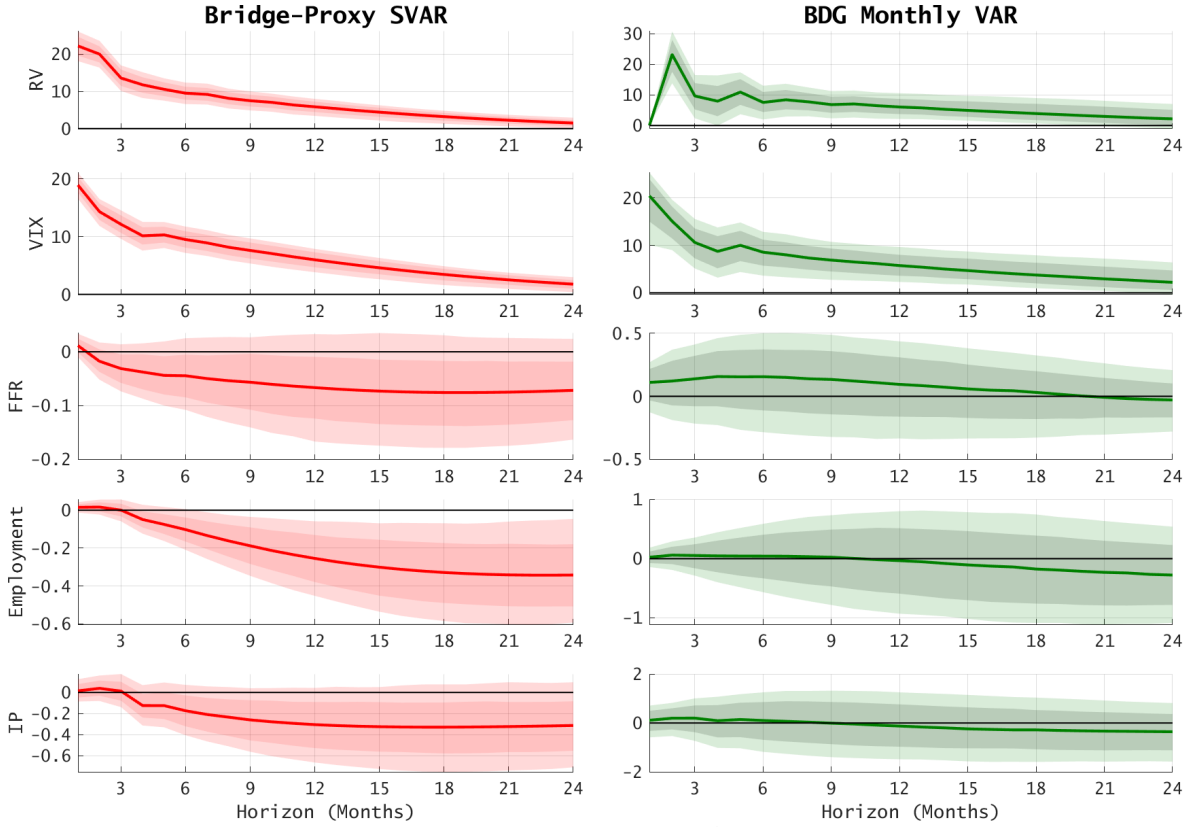


Figure 3: Comparison with BDG

IRFs to an uncertainty shock identified with the Bridge-PSVAR (red) and as in the BDG (green). The VAR includes $[rv, v_1, ffr, ip, emp]$ and is estimated in log-levels with the optimal number of lags (4) and includes a deterministic constant (the same specification as BDG). Light and dark shaded areas correspond respectively to 90% and 68% bootstrapped confidence bands, computed using 1000 wild bootstrap replications.

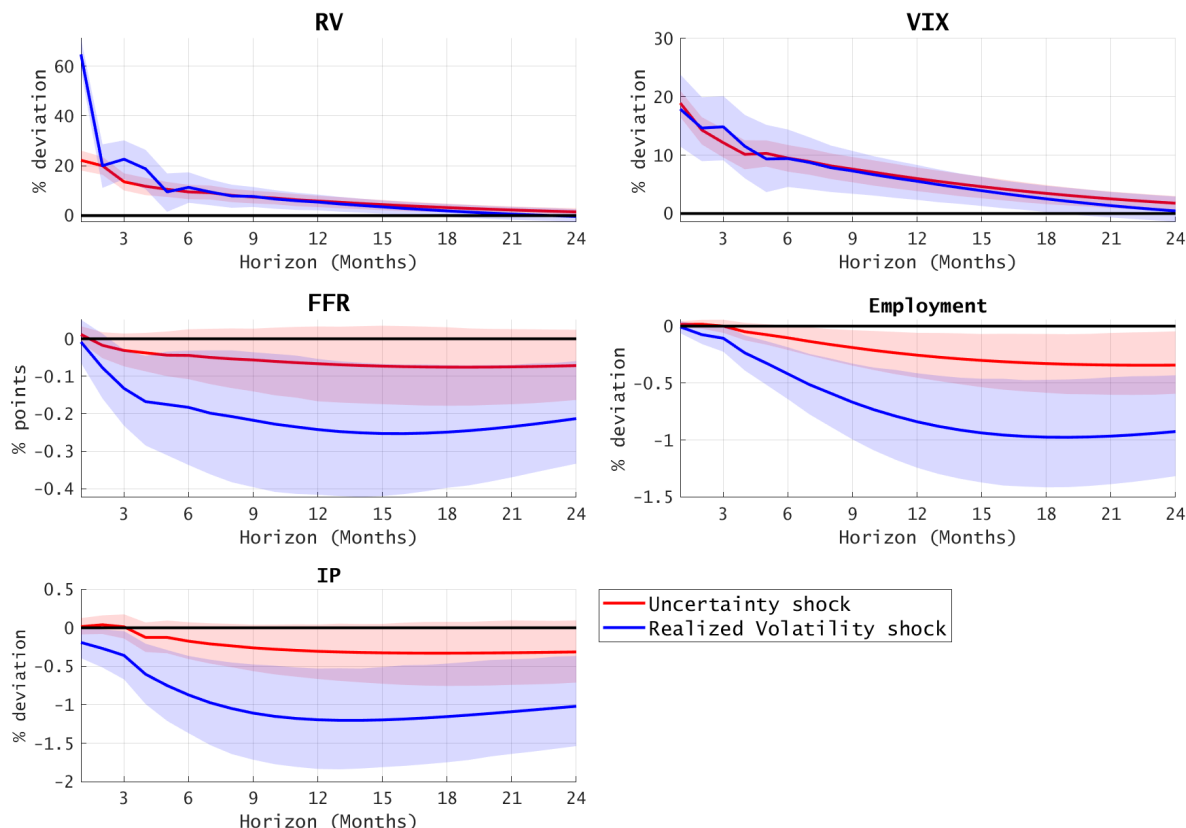


Figure 4: Comparison of IRFs: Uncertainty versus Realized Volatility Shocks

IRFs to an uncertainty (red) and to a realized volatility shock (blue) identified from our daily VAR and aggregated at the monthly frequency as averages. The VAR includes $[rv, v_1, ffr, ip, emp]$ and is estimated in log-levels with the optimal number of lags (4) and includes a deterministic constant (the same specification as BDG). Shaded areas correspond to 90% bootstrapped confidence bands, computed using 1000 wild bootstrap replications.

5.2 Monetary Policy Shocks

Recent works have documented that the monetary policy surprises (Gurkaynak et al., 2005) employed in the macro literature (Gertler and Karadi, 2015) are not proper monetary shocks because they are: i) correlated with previous macroeconomic conditions and contaminated with releases of information from the central bank (Ramey, 2016, Karadi and Jarocinski, 2018, and Miranda Agrippino and Ricco, 2018); and ii) are predictable using past equity returns (Neuhierl and Weber, 2018). We apply the Bridge-Proxy SVAR to identify monetary policy shocks in a daily VAR and estimate their macroeconomic effects. We show that our daily VAR yields monetary policy shocks that

are orthogonal to past information (macro and financial) and can consequently be interpreted as shocks. The IRFs obtained from the Bridge-Proxy SVAR are in line with the consensus in the literature. The results of this ancillary exercise are reported in *Online Appendix* for ease of exposition.

6 Conclusions

High-frequency identification has been recently introduced in empirical macroeconomics by means of event-studies. However, this approach can be applied only in particular cases and does not necessarily allow the researcher to fully isolate an economic shock of interest. In this paper we develop a novel methodology, the Bridge Proxy-SVAR, that identifies a shock of interest in a high-frequency VAR by employing the identification approaches developed in the SVAR literature. This allows us to isolate shocks of interest in a very general framework. The resulting innovations are aggregated as averages and used as proxies for the structural shocks in a VAR estimated at lower frequencies. This procedure allows for the identification of the shock of interest by imposing weaker assumptions compared to the traditional monthly/quarterly VAR commonly employed in the literature. Furthermore, this methodology does not mix the real innovations with the endogenous response of the system to the shocks because it employs the correct information set of agents.

Can structural analysis be performed by modeling disjointedly variables sampled at different frequencies? Yes. The positive answer to this question, and thus the validity of our methodology, rests upon both econometric propositions and Monte Carlo experiments. The Bridge Proxy-SVAR can correctly recover the impact effect of the shock of interest. We illustrate the usefulness of the Bridge Proxy-SVAR with an empirical application on the dynamic effects of uncertainty shocks. This empirical analysis

highlights that the same identification scheme may lead to different conclusions if applied at two different frequencies. Uncertainty shocks identified at the monthly frequency as innovations in expected volatility that have no impact on realized volatility have null macroeconomic effects. However, once the same identification strategy is shifted to a daily frequency, uncertainty shocks are recessionary and the monthly restriction is not supported by the data. Thus, the Bridge-PSVAR is particularly promising to improve SVARs analysis that can exploit information from financial markets and other daily sources, e.g. the macroeconomic effects of financial shocks.

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Appendix

A Alternative Definition of LF-IRFs

In this Section, we show how the HF shocks should be aggregated to the low-frequency if one employs the definition of LF-IRFs based on the temporal aggregation filters, which are specific to the particular temporal aggregation scheme applied. *Proposition IIA* is then employed in the proof of *Proposition I*.

A.1 Skip-Sampling

The skip-sampling case is simpler because the second step of the temporal aggregation process consists of the trivial filter $W(L) = \mathcal{I}$ that leaves the variables unaffected. Skip-sampling is usually applied by taking the last value: for example the last daily observation within the month. We focus on this skip-sampling scheme without loss of generality.^{1A} Eq.(10) is modified by temporal aggregation as:

$$D(L)A(L)y_t = D(L)B\varepsilon_t \quad (\text{A.1})$$

Under skip-sampling, the impact effect of ε on y is trivially given $\Theta_{0,\tau} = \Theta_{0,t} = B$. Suppose that the HF shocks are identified under the assumptions described in Equation 12.

Proposition IA. *Given an underlying HF-VAR temporally aggregated via skip-sampling, the IRF $\Theta_{0,\tau} = B$ can be recovered by projecting the reduced form residuals estimated from the LF-VAR, u_τ , on the last HF shock within the LF period. Thus the correct filter $J(L)$ applied to ε_t is $J(L) = \mathcal{I}$ such that $\varepsilon_\tau = \varepsilon_t$ for $\{\tau, t\} = m, 2m, \dots, T$.*

^{1A}Notice that the same results that we provide hold simply by using the shock corresponding to the skip-sampling scheme (e.g. take the first shock if skip-sampling is performed using the first HF value)

Proof: The correct impact matrix can be recovered simply projecting u_τ on $\varepsilon_{t,m-1}$. The LF reduced form residuals are given by:

$$u_\tau = D(L)u_t = D(L)B\varepsilon_t \quad (\text{A.2})$$

Recall that $D(L) = \mathcal{I} + D_1L + D_2L^2 + \dots + D_{pm-p}L^{pm-p}$ always contains the identity matrix as first term. Thus

$$D(L)B\varepsilon_t = B\varepsilon_t + D_1B\varepsilon_{t-1} + D_2B\varepsilon_{t-2} + \dots + D_{pm-p}B\varepsilon_{t-pm+p} \quad (\text{A.3})$$

Independently of the values of D_i (that depend on the HF-VAR lag length p and frequency mismatch m) and considering that ε_t are uncorrelated, the following relationship holds:

$$Proj(u_\tau/\varepsilon_t) = B = \Theta_{0,\tau} \quad (\text{A.4})$$

A.2 Averaging

The averaging case is more complex as the second filter is $W(L) = \mathcal{I} + L + L^2 + \dots + L^{m-1}$. We use the summing filter, which is equivalent to averaging up to a constant. Consequently, the system becomes

$$D(L)W(L)A(L)y_t = D(L)W(L)B\varepsilon_t \quad (\text{A.5})$$

The HF impact effect is again $\Theta_0^t = B$ but the IRFs must be consistently temporally aggregated if we want to dispose of a reliable metric of comparison. Under linearity, the impact effect of ε on y at low-frequency is given by $\Theta_0^\tau = \Theta_0^t + \Theta_1^t + \dots + \Theta_{m-1}^t$.

Proposition IIA. *Given an underlying HF-VAR temporally aggregated via averaging, the IRF*

$\Theta_{0,\tau}$ can be recovered by projecting the reduced form residuals estimated from the LF-VAR, u_τ , on the first HF shock within the LF period. Thus, the correct filter $J(L)$ applied to ε_t is $J(L) = L^{m-1}$ such that $\varepsilon_\tau = \varepsilon_{t-m+1}$ for $\{\tau, t\} = m, 2m, \dots, T$.

Proof: It is convenient to express the IRFs at horizon k employing the companion form of the VAR:

$$x_t = F x_{t-1} + \eta_t \quad (\text{A.6})$$

where

$$F = \begin{bmatrix} A_1 & A_2 & A_3 & \dots & A_p \\ \mathcal{I} & 0 & 0 & \dots & 0 \\ 0 & \mathcal{I} & 0 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & \mathcal{I} & 0 \end{bmatrix} \quad (\text{A.7})$$

$$\eta_t = \begin{bmatrix} u_t = B\varepsilon_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (\text{A.8})$$

$x_t = [y_t \ y_{t-1} \ \dots \ y_{t-p+1}]'$ where p is the lag length of the VAR and $x_{t-1} = Lx_t$, such that x_t is a vector of $(n \times p)$ variables. Then, considering that $\Theta_{0,t} = B$, the dynamic effects can be written as well in companion form as:

$$\tilde{\Theta}_{k,t} = F \tilde{\Theta}_{k-1,t} \quad k \in \mathbb{N} \quad (\text{A.9})$$

We are interested in the matrix that contains the impulse response at horizon k ,

positioned in $\tilde{\Theta}_{k,t}(1, 1) = \Theta_{k,t}$.

Kalman (1982) and **Kilic (2007)** have shown that this formulation of the IRFs corresponds to a generalized Fibonacci sequence of order p in the matrices $A_1, A_2, \dots, A_p$ (also referred to as generalized order p Fibonacci polynomial), denoted by $S_p(A_1, \dots, A_p)$. Given an initial condition B , $S_p(A_1, \dots, A_p)$ generates a sequence whose elements are a linear combination of the previous terms in the sequence weighted by $A_1, \dots, A_p$. A_1 is the weight associated to the previous element of the sequence whereas A_p multiplies the p -th previous element. For instance $S_0 = B$, $S_1 = A_1 S_0$, $S_2 = A_1 S_1 + A_2 S_0$, and so on and so forth.

Following **Marcellino (1999)**, we define the following vector of matrices D^v and A^v with dimension $1 \times pm$, where m denotes the frequency mismatch, and the matrix of matrices G with dimension $(pm - p) \times pm$

$$\begin{aligned}
D^v &= (D_1, D_2, \dots, D_{pm-p}) \\
A^v &= (A_1, A_2, \dots, A_p, 0, \dots, 0) \\
G &= \begin{bmatrix}
-\mathcal{I} & A_1 & A_2 & \dots & A_{p-1} & A_p & 0 & 0 & \dots & & 0 \\
0 & -\mathcal{I} & A_1 & A_2 & \dots & A_{p-1} & A_p & 0 & 0 & \dots & 0 \\
0 & 0 & -\mathcal{I} & A_1 & A_2 & \dots & A_{p-1} & A_p & 0 & 0 & \dots & 0 \\
\vdots & 0 & 0 & -\mathcal{I} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
& \vdots & 0 & \ddots & \ddots & & & & & & & \\
& & \vdots & \ddots & & & & & & & & \\
& & & & & & & & & A_{p-1} & A_p & 0 \\
0 & 0 & & \dots & \dots & 0 & -\mathcal{I} & A_1 & A_2 & \dots & A_{p-1} & A_p
\end{bmatrix}
\end{aligned}$$

Marcellino (1999) shows that $D(L)$ exists if $|G_{-k}| \neq 0$ where G_{-k} corresponds to the G matrix whose columns multiple of m (i.e. $m, 2m, \dots$) have been deleted. Then, the

coefficients of $D(L)$ are given by $D^v = -A_{-k}^v (G_{-k})^{-1}$. The intuition is that they have to be such that only the powers of L^m can have a coefficient different from 0 (since the variables are unobserved between L^m and L^{2m}).

Since our goal is the identification of the impact effect Θ_0^τ , we focus exclusively on the upper square block of G of dimension $(m-1) \times (m-1)$, denoted by $\tilde{G}$. Consistently, we denote $\tilde{A}^v$ and $\tilde{D}^v$ the vector containing the first $m-1$ elements of the original A^v and D^v vectors. The matrix $\tilde{G}$ is a very special matrix, being a Toeplitz upper triangular matrix with main diagonal $-\mathcal{I}$. [Sahin \(2018\)](#) shows that, if invertibility is satisfied, the inverse of this class of matrices, denoted in our case by $\tilde{G}^{-1}$, contains the elements of the Fibonacci sequence in the matrix $A_1, \dots, A_p$. By considering that

$$\tilde{D}^v = -\tilde{A}^v \tilde{G}^{-1}$$

it follows that the elements of $\tilde{D}^v$ correspond to $S_p(A_1, \dots, A_p)$, the Fibonacci sequence in $A_1, \dots, A_p$. Disregarding the initial effect B , the IRFs $\Theta_0^t, \Theta_1^t, \dots, \Theta_{m-1}^t$ and the elements of the temporal aggregation filter $\mathcal{I}, D_1, D_2, \dots, D_{m-1}$ are generated by the same generalized Fibonacci sequence $S_p(A_1, \dots, A_p)$. Finally, the temporally aggregated residuals are given by $u_\tau = D(L)W(L)\varepsilon_t$. The filter $W(L) = \mathcal{I} + L + \dots + L^{m-1}$ implies the first shock within the LF period, i.e. $\varepsilon_{t,1}$, enter u_τ through all the terms $\Theta_{0,t}, \Theta_{1,t}, \dots, \Theta_{m-1,t}$ and thus recovers the correct $\Theta_{0,\tau} = \Theta_{0,t} + \Theta_{1,t} + \dots + \Theta_{m-1,t}$. Thus, the projection of u_τ on ε_{t-m+1} yield the correct IRFs $\Theta_{0,\tau}$:

$$Proj(u_\tau / \varepsilon_{t-m+1}) = \Theta_{0,\tau} \tag{A.10}$$

■

B Proofs of the Propositions included in Section 3.3

This Section contains the proofs of Propositions I-II and Lemma I included in Section 3.3.

B.1 Proof of Proposition I

This proof follows directly from the proof of *Proposition IIA*, which showed that the IRFs $\Theta_0^t, \Theta_1^t, \dots, \Theta_{m-1}^t$ and the elements of the temporal aggregation filter $\mathcal{I}, D_1, D_2, \dots, D_{m-1}$ are generated by the same generalized Fibonacci sequence $S_p(A_1, \dots, A_p)$ (for the given initial condition B). This implies that the contemporaneous LF impact effect of the shock ε_{t-m+1} is given by the sum of the first m elements of the Fibonacci sequence denoted by S_p^{m-1} . The effect of ε_{t-m+2} is given by the sum of the elements in S_p^{m-2} and so on and so forth until the last HF shock ε_t that impacts through $B = S_p^0$. In the case of skip-sampling, $\Theta_{0,\tau} = S_p^0$ whereas in the case of averaging $\Theta_{0,\tau} = \sum_{i=1}^m B S_p^i$. In both cases, projecting u_τ on ε_τ recovers $\Theta_{0,\tau}$.

Consider that $\varepsilon_\tau = (I + L + \dots + L^{m-1}) \varepsilon_t$ and $u_\tau = D(L)W(L)B\varepsilon_t$ AV: I think there was a mistake in the ordering between $D(L)$ and $W(L)$ matrices in this equation where the first m elements of $D(L)$ are $I + D_1L + \dots + D_{m-1}L^{m-1}$ and $W(L) = (I + L + \dots + L^{m-1})$. By the result in *Proposition IIA* on the equivalence between the recursive definition of IRFs and the matrices $\mathcal{I}, D_1, D_2, \dots, D_{m-1}$, it is straightforward to verify that:

$$\begin{aligned} Proj(u_\tau/\varepsilon_\tau) &= \mathbb{E} \left[(\varepsilon_\tau \varepsilon_\tau)^{-1} \right] \mathbb{E} [\varepsilon_\tau u_\tau] \\ &= \sum_{j=0}^{m-1} \Gamma_j = \Theta_{0,\tau} \end{aligned}$$

■

B.2 Proof of Proposition II

Under standard assumptions, the Wold Representation Theorem implies that the innovations $\{\eta_t\}$ of a time series process $\{y_t\}$ are white noise. If the underlying process is well approximated by a $VAR(p)$, then this property extends to the residuals estimated by the VAR $\{u_t\}$ (Canova, 2007 Ch.4). Thus, it holds that:

$$\begin{aligned}\mathbb{E} \left[u_t u'_{t-j} \right] &= \Sigma \quad \text{for } j = 0 \\ \mathbb{E} \left[u_t u'_{t-j} \right] &= 0 \quad \text{for } \forall j \neq 0\end{aligned}\tag{A.11}$$

Property A.11 extends to the structural shocks $\varepsilon_t = B^{-1}u_t$ because they are a linear transformation of u_t : $\mathbb{E} \left[\varepsilon_t \varepsilon'_{t-j} \right] = 0$ for $j = 1, \dots, p$. Furthermore, it follows from identification that $\mathbb{E} [\varepsilon_t \varepsilon_t] = 0$. Without loss of generality, partition $\varepsilon_t = \begin{bmatrix} \varepsilon_t^1 & \varepsilon_t^{\bar{1}} \end{bmatrix}'$ and define $\varepsilon_t^1 = b_{\bullet 1} u_t$ and $\varepsilon_t^{\bar{1}} = b_{\bullet \bar{1}} u_t$, where $b_{\bullet 1}$ denotes the first column of the impact matrix B and $b_{\bullet \bar{1}}$ the remaining columns. Combining the previous properties, it holds

$$\mathbb{E} \left[\varepsilon_t^1 \varepsilon_{t-j}^{\bar{1}'} \right] = 0 \quad \text{for } \forall j\tag{A.12}$$

Consider now the aggregated shocks $\varepsilon_\tau = \sum_{i=0}^{m-1} \varepsilon_{\tau-i}$. To evaluate $\mathbb{E} \left[\varepsilon_\tau^1 \varepsilon_\tau^{\bar{1}'} \right]$, we need to compute the correlations between all the elements that are summed into ε_τ^1 and $\varepsilon_\tau^{\bar{1}}$, which are all null by (A.12). Thus, $\mathbb{E} \left[\varepsilon_\tau^1 \varepsilon_\tau^{\bar{1}'} \right] = 0$. ■

B.3 Proof of Lemma I

Notice that $\varepsilon_{1t} = b_{1\bullet} u_t$ and $\varepsilon_{2t} = b_{2\bullet} u_t$ are two structural shocks obtained as linear combination of the residuals $u_{t,\cdot}$, where $b_{1\bullet}$ denotes the first column of the impact matrix B and $b_{2\bullet}$ the remaining columns. Based on Proposition II, $\mathbb{E} \left[u_t u'_{t-j} \right] = 0$ for $\forall j \neq 0$. This extends to $\mathbb{E} \left[\varepsilon_{1t} \varepsilon_{it-j}' \right]$ for $\forall j \neq 0$ and $i = 1, 2$ by the property of the linear operator

$b_{1\bullet}$. Plus, $\mathbb{E}[\varepsilon_{1t}\varepsilon_{2t}] = 0$ since they are structural shocks. Thus, each element of the sum in $\varepsilon_{1\tau}$ is uncorrelated to the elements in $\varepsilon_{2\tau}$ and so it is their sum. Based on Proposition II, the lack of autocorrelation of order 1 holds. ■